

7.5 Modelling Data Using Logarithmic Functions

EXAMPLE 1 Using logarithmic regression to solve a problem graphically

The flash on most digital cameras requires a charged capacitor in order to operate. The percent charge, Q , remaining on a capacitor was recorded at different times, t , after the flash had gone off.

The $t.5$ flash duration represents the time until a capacitor has only 50% of its initial charge. The $t.5$ flash duration also represents the length of time that the flash is effective, to ensure that the object being photographed is properly lit.

- Construct a scatter plot for the given data. $a = 0.4596557$
- Determine a logarithmic model for the data. $b = -0.0998423$
- Use your logarithmic model to determine the $t.5$ flash duration to the nearest hundredth of a second.

Percent Charge, Q (%)	Time, t (s)
100.00	0
90.26	0.01
73.90	0.03
60.51	0.05
49.54	0.07
40.56	0.09

$$y = 0.4596557 - 0.0998423 \ln x$$

c) $\boxed{2^{\text{nd}}}$ $\boxed{\text{calc}}$ $\boxed{\text{value}}$ $x = 50$

$$y = 0.069 \text{ seconds}$$

often on a log graph, the dependent and independent variable are switched

* after 0.2 seconds, what percentage charge remains on the capacitor?

graph $y = 0.2$ and find P.O.T. $\Rightarrow Q = 13.5\%$

Your Turn

The $t.1$ flash duration represents the time until a capacitor has just 10% of its initial charge. Determine the $t.1$ flash duration for the data above, to the nearest hundredth of a second.

$$y = 0.23 \text{ seconds}$$

EXAMPLE 2

Using logarithmic regression to solve a problem algebraically

Caffeine is found in coffee, tea, and soft drinks. Many people find that caffeine makes it difficult for them to sleep. The following data was collected in a study to determine how quickly the human body metabolizes caffeine. Each person started with 200 mg of caffeine in her or his bloodstream, and the caffeine level was measured at various times.



- Determine the equation of the logarithmic regression function for the data representing time as a function of caffeine level.
- Determine the time it takes for an average person to metabolize 50% of the caffeine in her or his bloodstream. Round your answer to the nearest tenth of an hour.
- Paula drank a cup of coffee that contained 200 mg of caffeine at 10:00 a.m. How much caffeine will be in her bloodstream at 9:00 p.m. that evening? Round your answer to the nearest milligram.

$$a) y = 44.45369223 - 8.370711434 \ln x$$

$$x_{\min} = 0 \quad y_{\min} = 0$$

$$x_{\max} = 200 \quad y_{\max} = 22$$

$$x_{sd} = 10 \quad y_{sd} = 2$$

b) 50% of the caffeine is metabolized after 5.9 hrs

$$50\% \text{ of } 200 \text{ mg} = 100 \text{ mg}$$

c) 54 mg

Caffeine Level in Bloodstream, m (mg)	Time after Ingesting, t (h)	Caffeine Level in Bloodstream, m (mg)	Time after Ingesting, t (h)
168	1.0	33	14.0
167	1.5	80	7.5
113	5.0	145	3.0
145	3.0	100	6.0
90	6.5	71	8.5
125	4.0	156	2.0
138	3.5	153	2.5
77	8.0	130	4.0
83	7.0	90	6.5
50	12.0	112	5.0
150	2.5	32	16.0
55	12.0	23	18.0
112	5.0	25	17.5
84	7.0	45	13.0
136	3.5	27	18.5
180	1.0	18	20.0
110	5.0	29	15.0
75	8.0	43	12.0
76	9.0	25	17.5
49	12.5	21	19.0