7.5 Solving Linear Trigonometric Equations

Key points: Look for two things: size of reference angle, and location (CAST rule)

"SIMPLIFY" the angular argument (in brackets) before solving
** But you need to DE-SIMPLIFY afterwards**

Remember that there are infinite solutions to a trig function!!

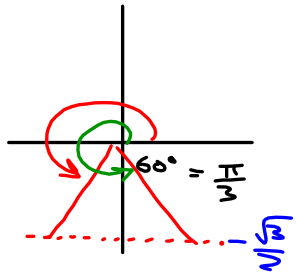
Example 1:

$$\sin x = -\frac{\sqrt{3}}{2}, \quad 0 \le x \le 2\pi$$

answer in radians on $[0, 2\pi]$

$T = \frac{2\pi}{1} = 2\pi$

soln in QIII & QIV



$x = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$x = \frac{4\pi}{3}$; $\frac{5\pi}{3}$

IN General:

$$x = \frac{4\pi}{3} + 2\pi n$$
$$x = \frac{5\pi}{3} + 2\pi n$$
$$n \in I$$

Example 2:

argument

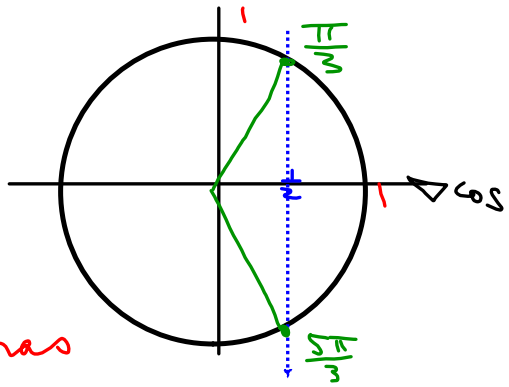
$$2\cos\left(1x-\frac{\pi}{6}\right)=1,\quad 0\le x\le 2\pi$$

$T = \frac{2\pi}{1} = 2\pi$

Solve for x

$\cos\left(x-\frac{\pi}{6}\right)=\frac{1}{2}$

$\left(x-\frac{\pi}{6}\right)=\cos^{-1}\left(\frac{1}{2}\right)$



but this has 2 solns

$x-\frac{\pi}{6}=\frac{\pi}{3}$

$x=\frac{2\pi}{6}+\frac{\pi}{6}$

$x=\frac{3\pi}{6}$

$x=\frac{\pi}{2}$

$x-\frac{\pi}{6}=\frac{5\pi}{3}$

$x=\frac{10\pi}{6}+\frac{\pi}{6}$

$x=\frac{11\pi}{6}$

general: $x=\frac{\pi}{2}+2\pi n$, $x=\frac{11\pi}{6}+2\pi n$ $\Big\}\ n\in I$

Example 3:

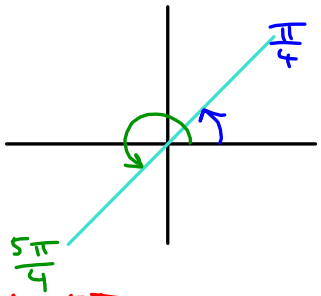
NB: T for tan is π

$\therefore T = \frac{\pi}{2}$

$$1 - \tan\left(2x + \frac{\pi}{2}\right) = 0, \quad 0 \le x \le 2\pi$$

$$+\tan\left(2x + \frac{\pi}{2}\right) = +1$$

$$\left(2x + \frac{\pi}{2}\right) = \tan^{-1}(1)$$



but $\tan^{-1}(1) = \frac{\pi}{4}$ and $\frac{5\pi}{4}$

$2x + \frac{\pi}{2} = \frac{\pi}{4}$

$2x = \frac{\pi}{4} - \frac{2\pi}{4}$

$2x = -\frac{\pi}{4}$

not on interval!! $x = -\frac{\pi}{8}$

$2x + \frac{\pi}{2} = \frac{5\pi}{4}$

$2x = \frac{5\pi}{4} - \frac{2\pi}{4}$

$2x = \frac{3\pi}{4}$

$x = \frac{3\pi}{8}$

but I can add multiples of the period $(\frac{\pi}{2})$ to get all sol^{n}s on $[0, 2\pi]$

So $x = -\frac{\pi}{8} + \frac{\pi}{2} = \frac{3\pi}{8}$ same!

$x = \frac{3\pi}{8} + \frac{\pi}{2} = \frac{7\pi}{8}$

$x = \frac{7\pi}{8} + \frac{4\pi}{8} = \frac{11\pi}{8}$

$x = \frac{11\pi}{8} + \frac{4\pi}{8} = \frac{15\pi}{8}$

general: $\frac{3\pi}{8} + \frac{\pi}{2}n$, $n \in \mathbb{I}$

Example 4:

When a projectile leaves a starting point at an angle of elevation of θ with velocity v, the horizontal distance it travels is determined by:

$$d = \frac{v^2}{32}\sin 2\theta$$

$T = \frac{2\pi}{2}$

$T = \pi$

Where d is measured in feet and v in feet per second.

An outfielder throws a ball at a speed of 75 miles per hour to the catcher who is 200 feet away. At what angle was the ball thrown?

Note: there are 5280 feet in a mile

$$75\ \frac{\text{mi}}{\text{hr}} \times \frac{1\ \text{hr}}{3600\ \text{s}} \times \frac{5280\ \text{ft}}{1\ \text{mi}} = 110\ \text{ft/s}$$

plug into equⁿ:

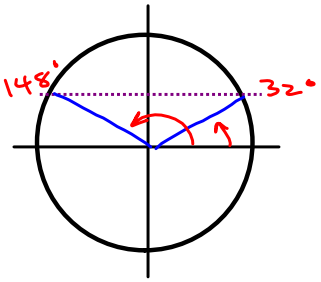
$$200 = \frac{(110)^2}{32}\sin 2\theta$$

$$\frac{200(32)}{110^2} = \sin 2\theta$$

$$0.5289 = \sin 2\theta$$

$$2\theta = \sin^{-1}(0.5289)$$

has 2 solns $\Rightarrow$ 32° and 148°



$$\frac{2\theta}{2} = \frac{32^\circ}{2} \qquad \frac{2\theta}{2} = \frac{148^\circ}{2}$$

$\theta \doteq 16^\circ$ $\qquad$ $\theta = 74^\circ$

but $T = \pi$ so we must add 180° to each answer!!!

$\therefore \theta = 16^\circ + 180^\circ = 196^\circ$

or $\theta = 74^\circ + 180^\circ = 254^\circ$

have no real meaning!

$\therefore$ the ball is thrown @ 16° or 74°

Homefun:

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