7.6 Solving Quadratic Trigonometric Equations

Solution strategies:

1. common factor
2. trinomial factor $\rightarrow x^2 + 3x + 2$
   $= (x+2)(x+1)$
   $\hookrightarrow$ difference of squares
3. Quad. Formula
4. Use an identity
   then go back to 1, 2 & 3

NB Collect all terms on one side (other side $= 0$) first!

Solve for θ, $0 \le \theta \le 2\pi$ rads

$$\tan \theta \cos^2 \theta = \tan \theta$$

$$\tan \theta \cos^2 \theta - \tan \theta = 0$$

$$\tan \theta (\cos^2 \theta - 1) = 0$$

$\tan \theta = 0$ $\cos^2 \theta - 1 = 0$

$\tan \theta = \dfrac{\sin \theta}{\cos \theta}$ $\sqrt{\cos^2 \theta} = \pm \sqrt{1}$ π

$\therefore \sin \theta = 0$ $\cos \theta = \pm 1$

$\boxed{\theta = 0, \pi, 2\pi}$ $\cos \theta = 1 \qquad \cos \theta = -1$

$\boxed{\theta = 0, 2\pi} \qquad \boxed{\theta = \pi}$
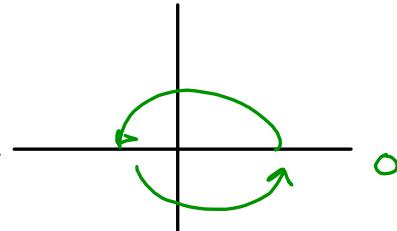
$\boxed{\text{So, on } [0, 2\pi], \ \theta = 0, \pi, 2\pi}$

general sol^n $\theta = \pi n, \ n \in I$

Example 2:

Solve for x, $0 \le x \le 2\pi$

$$3\sin^2 x - \sin x - 2 = 0$$

$(3\sin x + 2)(\sin x - 1) = 0$

if you want

Let $y = \sin x$

$3y^2 - y - 2 = 0$

$(3y + 2)(y - 1) = 0$

$3\sin x = -2$ $\sin x = 1$

$\sin x = -\frac{2}{3}$ $\boxed{x = \frac{\pi}{2}}$

$\boxed{\begin{array}{l} x \doteq -0.73 \\ \text{not in interval!} \end{array}}$

$x \doteq -0.73 + 2\pi$

$\boxed{x = 5.55}$

$x = \pi + 0.73$

$\boxed{x = 3.87}$

$\theta = 0.73$
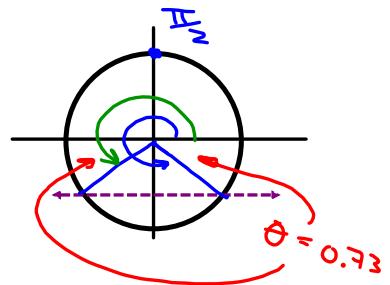
general :

$x = \frac{\pi}{2} + 2\pi n$

$x = 5.55 + 2\pi n$ $\Big\} n \in I$

$x = 3.87 + 2\pi n$

No interval ... So general sol'n !!!

Example 3:

Solve for x

$$-10\cos^2 x - 3\sin x + 9 = 0$$

N.B. can't factor with $\sin x$ & $\cos x$ ...

need $\sin^2 + \cos^2 = 1$

$\cos^2 = \boxed{1 - \sin^2}$

$$-10(1 - \sin^2 x) - 3\sin x + 9 = 0$$

$$-10 + 10\sin^2 x - 3\sin x + 9 = 0$$

$$10\sin^2 x - 3\sin x - 1 = 0$$

could sub $y = \sin x$

$$(2\sin x - 1)(5\sin x + 1) = 0$$

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\frac{5\pi}{6} = 150°$ $\frac{\pi}{6} = 30°$

$x = 30°, 150°$

$\boxed{\begin{array}{l} x = 30° + 360°n \\ x = 150° + 360°n \end{array}}$ $n \in I$

$5\sin x = -1$

$\sin x = -\frac{1}{5}$
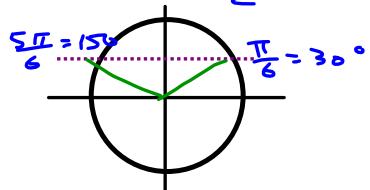
$x = -11.5°$

$180° + 11.5°$

$-11.5°$

$x = -11.5° + 360°$

$\boxed{x = 348.5°}$ $+ 360°n$

$x = 180° + 11.5°$

$\boxed{x = 191.5°}$ $+ 360°n$

$n \in I$
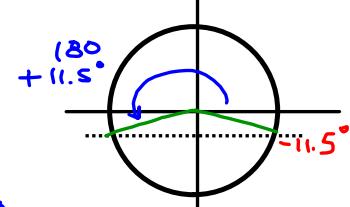
Example 4:

Solve for x, $-\pi \le x \le \pi$

$$\sec^2 x + 5\tan x = -2$$

$$\sec^2 x = 1 + \tan^2 x$$

$$1 + \tan^2 x + 5\tan x = -2$$

$$\tan^2 x + 5\tan x + 3 = 0$$

$$(\tan x + 1)(\tan x ? 3) = 0 \quad \text{not factorable!!}$$
$$\rightarrow \text{use Q.F.}$$

$$\tan x = \frac{-5 \pm \sqrt{5^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{13}}{2}$$

$$\tan x = -4.31 \qquad\qquad \tan x = -0.695$$

$$x = -1.34 \qquad\qquad\qquad x = -0.61$$

$$x = -1.34 + \pi \qquad\qquad x = -0.61 + \pi$$

$$x = 1.80 \qquad\qquad\qquad x = 2.53$$

−0.61
−1.31
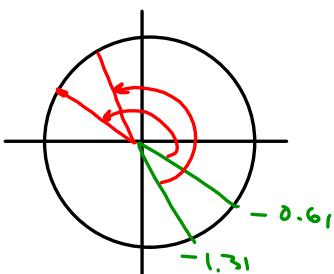
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