8.1 & 8.2 Introducing the Logarithmic Function

The logarithmic function is... the inverse of the exponential function.

$y = 2^x$ ⟶ exponential: the input is an exponent ← x

To find an inverse we... Switch x & y → solve for y algebraically / table of values / Domain & Range

$x = 2^y$ ⟶ give me a number (x) and I'll tell you what power to raise 2 to, to get it.

The "log" form of $x = 2^y$ is $y = \log_2 x$ ⇒ reads: "log x base 2"

exponential form: $b^y = x$ ⟺ logarithmic form: $y = \log_b x$

base, exponent

the value of the logarithm (it's an exponent), base, the argument

ex// $2^3 = 8 \Rightarrow 3 = \log_2 8$

ex// $x = \log_3 27 \Rightarrow 3^x = 27$

$\therefore x = 3$

In general... The inverse of $f(x) = b^x$ would be $f^{-1}(x) = \log_b x$

Logarithms are also functions

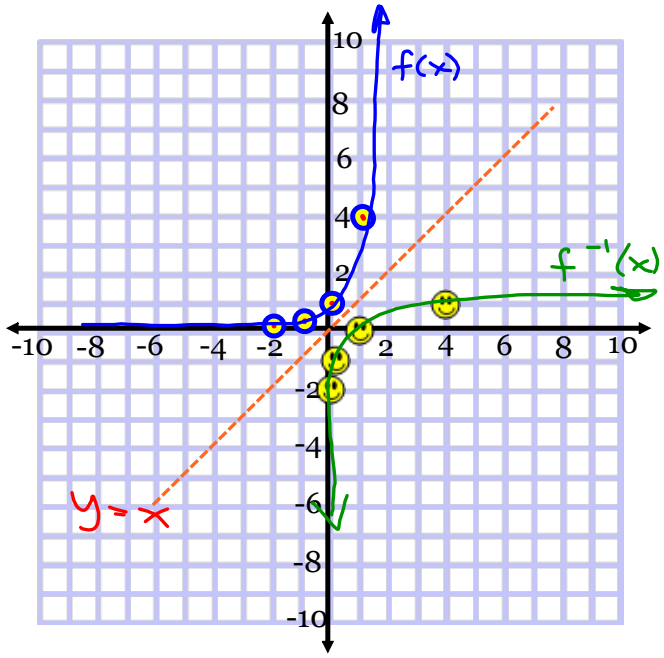
$3^{-2} = \frac{1}{3^2}$

Example:

The inverse of $f(x) = 4^x$ would be $f^{-1}(x) = \log_4 x$

$f(x) = 4^x$

x	y
-2	1/16
-1	1/4
0	1
1	4
2	16
3	64



$f^{-1}(x) = \log_4 x$

x	y
1/16	-2
1/4	-1
1	0
4	1
16	2

Practice: Complete the table

Exponential Form	Logarithmic Form
$2^3=8$	$\log_2 8 = 3$
$10^2=100$	$\log_{10} 100 = 2$
$\left(\frac{1}{2}\right)^4 = \frac{1}{16}$	$\log_{1/2}\left(\frac{1}{16}\right) = 4$
$3^{-4} = \frac{1}{81}$	$\log_3\left(\frac{1}{81}\right) = -4$
$2^5 = 32$	$\log_2 32 = 5$
$3^3 = 27$	$\log_3 27 = 3$
$8^{-2} = \frac{1}{64}$	$\log_8\left(\frac{1}{64}\right) = -2$

Graphing the Common Logarithmic Function

The common logarithmic function $y = \log_{10} x$ is often written as $y = \log x$ (no base implies "base 10")

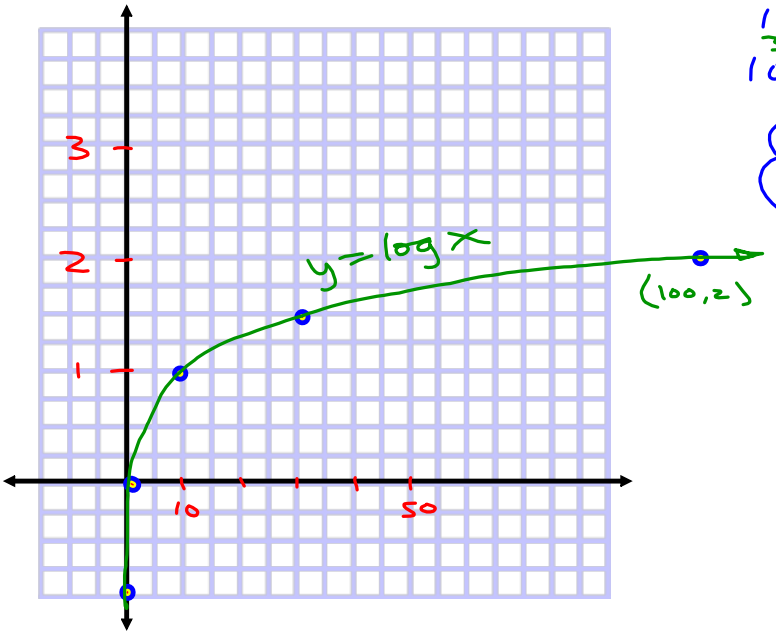
The key points of the common logarithmic function:

x	y
0.1	−1
1	0
10	1
32	1.5
100	2

$\log_{10} 0.1 = y$

$10^y = 0.1$

And the graph:



Function	Domain	Range	End Behaviour	Asymptotes	Intercepts
$y = \log x$	$x \in (0, \infty)$	$y \in \mathbb{R}$	$x \to \infty$, $y \to \infty$ $x \to 0$, $y \to -\infty$	V.A. of $x = 0$	$(1, 0)$

the V.A occurs where the argument = 0
ex// $y = \log(x+2)$ has V.A @ $x = -2$

As with other functions we can transform logarithmic functions.

A general transformed logarithmic function

$$y = a\log_{10}(k(x - d)) + c$$

vertical translation

vert. stretch/ reflection

horizontal stretch/ reflection

horizontal translation

Example 1: Graph $y = -2\log_{10}(x - 4)$

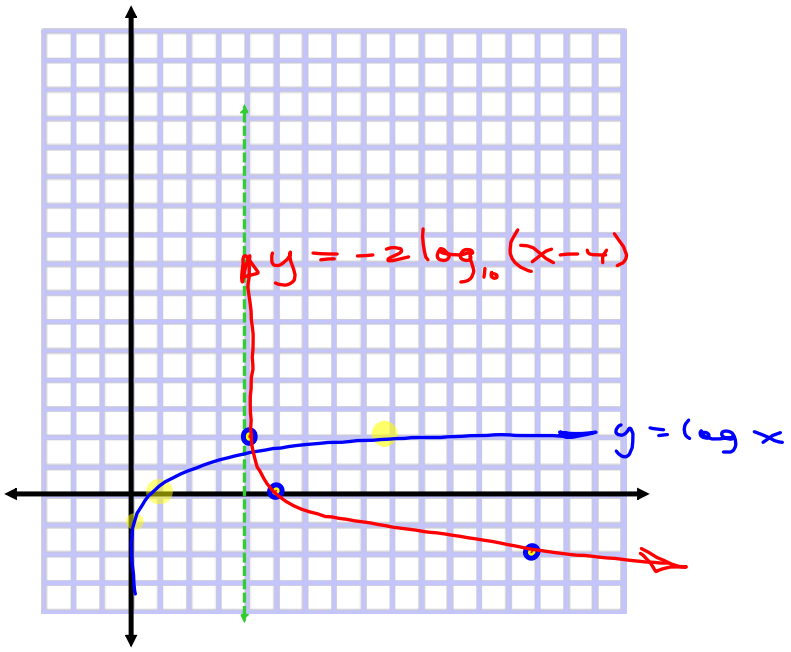
take it one step at a time

shift 4 units right

vert. stretch 2x

reflection in x-axis

	d = 4	x	y	a < 0	a = -2
	4.1	1/10	-1	1	2
	5	1	0	0	0
	14	10	1	-1	-2
	36	32	1.5	-1.5	-3
	104	100	2	-2	-4



What is the equation of the asymptote? $x = 4$

What is the domain? $\{x \in \mathbb{R} \mid x > 4\}$

What is the range? $\{y \in \mathbb{R}\}$

HOMEFUN:

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