

Chapter 8: Sinusoidal Functions

8.1 Understanding Angles

radian: the measure of the **central angle** of a circle subtended by an **arc** that is the same **length** as the **radius** of the circle.

standard position: when the initial arm is on the x-axis and the **vertex** is at the origin

initial arm: the arm of an angle in standard position that lies on the **positive** x-axis

terminal arm: the arm of an angle in standard position that meets the initial arm at the origin to form an angle

EXAMPLE 1 Expressing 1 radian in degrees

The measure of $\angle PCA$ is 1 radian. Calculate the measure of $\angle PCA$ in degrees.

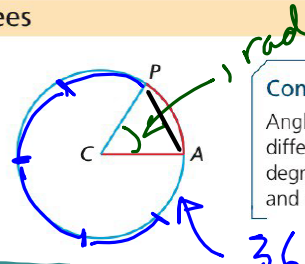
$$C = 2\pi r$$

$$\frac{360^\circ}{2\pi} = \frac{2\pi r}{2\pi}$$

$$\frac{180^\circ}{\pi} = r$$

$$180^\circ = \pi \text{ rad}$$

$$57.2^\circ = 1 \text{ rad}$$



Communication Tip
Angles can be measured using different units. These include degrees, radians, gradients, and minutes and seconds.

$$360^\circ = ? \text{ rad}$$

$$360^\circ \approx 6 \text{ rad}$$

$$1 \text{ rad} \approx 60^\circ$$

EXAMPLE 2 Estimating values of angles in radian measure

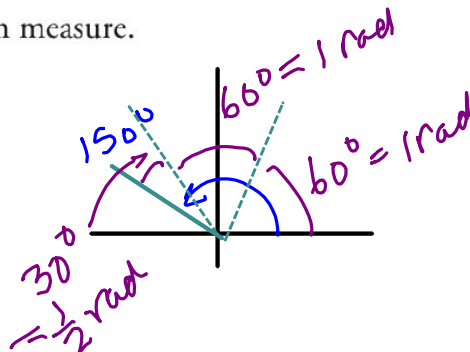
Estimate the value of each angle in radian measure.

- a) 90° b) 45° c) 150°

$$\approx 1.5 \text{ rad}$$

$$\approx 0.75$$

$$\approx 2.5$$



$$\pi \text{ rad} = 180^\circ$$

$$3.141592\dots = 180^\circ$$

exact relationship

EXAMPLE 3 | Estimating angles greater than 180° in radian measure

Estimate the value of each angle in radian measure.

$$\begin{array}{lll}
 \text{a) } 240^\circ & \text{b) } 450^\circ & \text{c) } 690^\circ = 720^\circ - 30^\circ \\
 \approx 4 & \approx 360^\circ + 90^\circ & \approx 12 \text{ rad} - 0.5 \text{ rad} \\
 & = 6 + 1.5 & = 11.5 \text{ rad} \\
 & = 7.5 \text{ rad} &
 \end{array}$$

Your Turn

Estimate the value of each angle in radian measure.

$$\begin{array}{ll}
 \text{a) } 120^\circ & \text{b) } 135^\circ \\
 = 2 \times 60^\circ & = 120^\circ + 15^\circ \\
 \approx 2 \text{ rad} & = 2 \text{ rad} + 0.25 \\
 & = 2.25 \text{ rad}
 \end{array}$$

Estimate the value of each angle in radian measure.

$$\begin{array}{lll}
 \text{a) } 420^\circ & \text{b) } 495^\circ & \text{c) } 660^\circ \\
 = 7 \times 60^\circ & \rightarrow 360^\circ + 135^\circ & \rightarrow 360^\circ + 300^\circ \\
 = 7 \text{ rad} & = 6 \text{ rad} + 2.25 & = 6 + 5 \\
 & = 8.25 & = 11 \text{ rad.}
 \end{array}$$

EXAMPLE 4 | Comparing angles in radian measure

Determine which angle is larger: 3π or 8.

$$\begin{array}{l}
 3\pi \\
 \approx 3 \times 3 \text{ rad} \\
 = 9 \text{ rad}
 \end{array}
 >
 \begin{array}{l}
 8
 \end{array}$$