8.2 & 8.3 Graphs of Periodic Functions

periodic function: a function whose graph repeats in regular intervals or cycles (or revolutions)

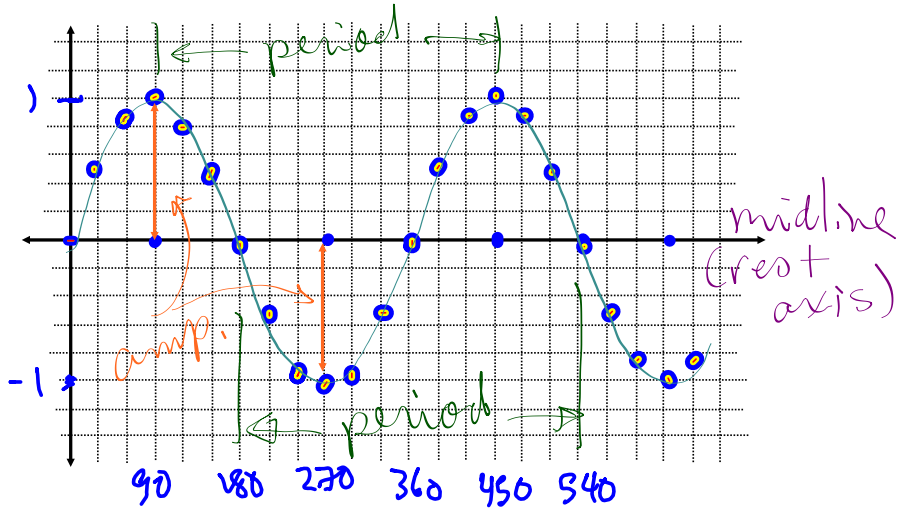
midline: the horizontal line halfway between the maximum and minimum values of a periodic function; also known as the rest axis

amplitude: the distance from the midline to either a max or min value of a periodic function; always expressed as a positive number

period: the length to complete one cycle

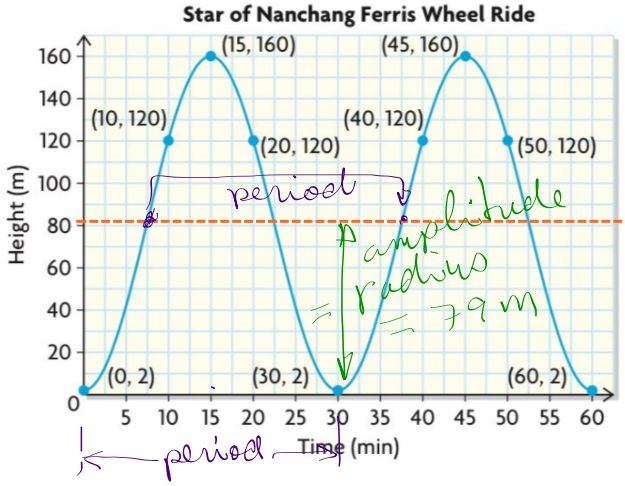
sinusoidal function: any periodic function whose graph has the same shape as that of ***y = sin x*** (also looks like $y = \cos x$)

Let's graph ***y = sin x*** using values of x that are multiples of 30º



x	sin x
0	0
30	0.5
60	0.867
90	1
120	0.867
150	0.5
180	0
210	-0.5
240	-0.867
270	-1
300	-0.867
330	-0.5
360	0

Identify the amplitude, period and midline



A. How can you tell, from Simone's graph, that the lowest part of the Ferris wheel is 2 m off the ground?

coordinates (0, 2) — time, height

B. Determine the maximum value of the graph. What is the height of the Ferris wheel?

160 m → $\{y \in \mathbb{R} \mid 2 \le h \le 160\}$ diameter — 158 m

C. Determine the range of the graph. What does this value represent?

D. Determine the amplitude of the graph. What does this value represent?

Amp = 79 m = radius

E. Determine the equation of the midline. What does this value represent?

→ $h = 81$ ⇒ height of the center

F. Determine the period of the graph. Explain your method.

→ 30 minutes : low to low

G. What length of time is needed for the Star of Nanchang to make one full revolution?

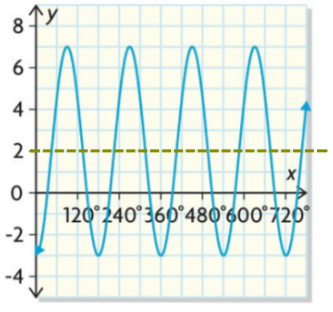
→ 30 minutes

H. How long does it take to get to the top of the Ferris wheel from the bottom?

→ $\frac{1}{2}$ of a period = 15 min

EXAMPLE 1 Describing the graph of a sinusoidal function in degree measure

The graph of a sinusoidal function is shown. Describe this graph by determining its range, the equation of its midline, its amplitude, and its period.



R: $\{y \in \mathbb{R} \mid -3 \le y \le 7\}$

top to bottom = 10

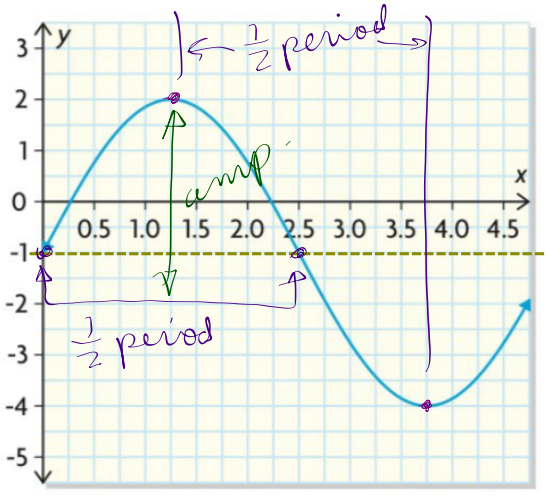
∴ amplitude = $\frac{10}{2} = 5$ ⇒ midline = min + amp.

$y = 2$

(period) $T = 180°$

EXAMPLE 2 Describing the graph of a sinusoidal function in radian measure

The graph of a sinusoidal function is shown. Describe this graph by determining its range, the equation of its midline, its amplitude, and its period.



midline = average value
$= \frac{\max + \min}{2}$
$= \frac{2 + (-4)}{2}$
$= \frac{-2}{2} = -1$

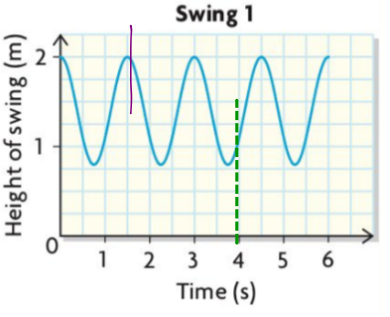
period = 2.5 × 2
= 5

amp = 3

$y \in \mathbb{R} \mid -4 \le y \le 2$

EXAMPLE 3 Connecting a sinusoidal function to oscillating motion

For a physics project, Morgan and Lily had to graph and analyze an example of simple harmonic motion. Morgan swung on a swing, and Lily used a motion detector to measure Morgan's height above the ground over time, as she swung back and forth. The girls then graphed their data as shown. At the end of each cycle, the swing returned to its initial position, which resulted in a sinusoidal graph.



a) Interpret the graph.
b) Determine Morgan's height above the ground at 4 s.

a) midline: $y = \frac{2 + 0.75}{2}$

$y = 1.375$

amplitude: $\frac{\max - \min}{2}$

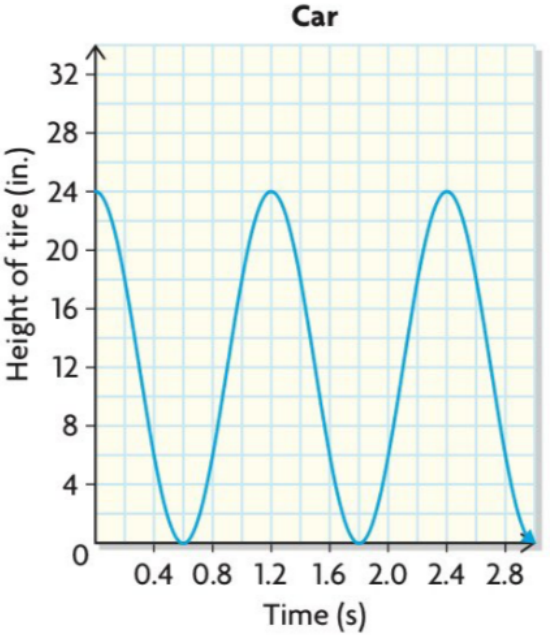
$= \frac{2 - 0.75}{2} = 0.625$

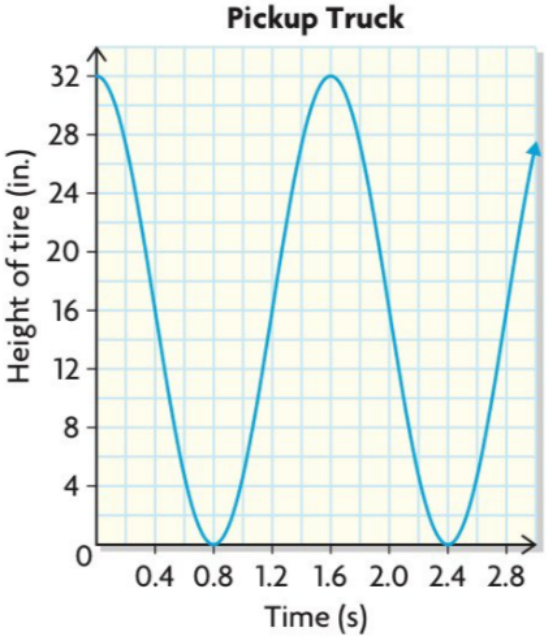
T = 1.5 sec.

b) h = 1 m

EXAMPLE 4 Comparing two sinusoidal functions

Alexis and Colin own a car and a pickup truck. They noticed that the odometers of the two vehicles gave different values for the same distance. As part of their investigation into the cause, they put a chalk mark on the outer edge of a tire on each vehicle. The following graphs show the height of the tires as they rotated while the vehicles were driven at the same slow, constant speed. What can you determine about the characteristics of the tires from these graphs?





Max values:	24	&	32
Min values:	0	&	0
Amplitude:	12	&	16
Diameter:	24	&	32
Midline:	h = 12	&	h = 16
Period:	1.2	&	1.6

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& Pg. 536 # 3-6, 10, 12, 13, 15