8.3 Evaluating Logarithms

$\{16^{1/4} = \sqrt[4]{16} = 2\}$

Helpful reminders: Exponent Rules

Zero exponent: $a^0 = 1 \longrightarrow \log_a 1 = 0$

Exponent of 1: $b^1 = b \longrightarrow \log_b b = 1$

Negative exponent: $b^{-a} = \frac{1}{b^a} \longrightarrow \log_b\left(\frac{1}{b^a}\right) = -a$

ex// $\log_3\left(\frac{1}{81}\right) = \log_3(3^{-4})$
$= -4$

Fractional Exponent: $b^{c/a} = \sqrt[a]{b^c}$ $\longrightarrow$ ex// $\log_9 3 = \log_9 \sqrt{9}$

$b^{c/a} = (b^c)^{1/a} = (b^{1/a})^c$ $= (\sqrt[a]{b})^c$

$9^? = \sqrt{9}$

$= \frac{1}{2}$

The Powers:

of 2: 2, 4, 8, 16, $\overset{2^5}{32}$, 64, 128, 256, 512, $\overset{2^{10}}{1024}$...

of 3: 3, 9, 27, 81, 243, 729, . . .

of 4: 4, 16, 64, 256, 1024, 4096, . . .

of 5: 5, 25, 125, 625, . . .

General strategy... write the argument as an exponential term with the same base...

Examples:

(a) $\log_4 64$ (b) $\log_3\left(\frac{1}{27}\right)$ (c) $\log_2(-4)$ (d) $\log_5 \sqrt[3]{25}$

(a)

$4^x = 64$

$4^x = (4)^3$

$\boxed{x=3}$

(b)

$3^x = \frac{1}{27}$

$3^x = \frac{1}{3^3}$

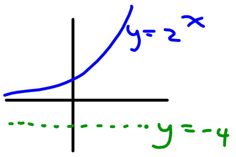
$3^x = 3^{-3}$

$\boxed{x=-3}$

(c)

$2^x = -4$

$2^x = -(2)^2$



no sol'n

(d)

$5^x = \sqrt[3]{25}$

$5^x = 25^{1/3}$

$5^x = (5^2)^{1/3}$

$5^x = 5^{2/3}$

$\boxed{x = 2/3}$

Interesting restrictions:

* the base of an exponential CANNOT be < 0

... try it: $y = (-2)^x \Rightarrow b < 0$

↳ same applies to a logarithm

* the base of an exponential $\neq 1$

$y = 1^x$ is linear $\therefore b \neq 1$

↳ Same for logs

Calculating "Nasty" Logarithms

Example: $\log_5 27 = x$

$\boxed{5^x = 27}$

① guess n' check

hmm... close to 2

try 2.1 ; $5^{2.1} = 29$

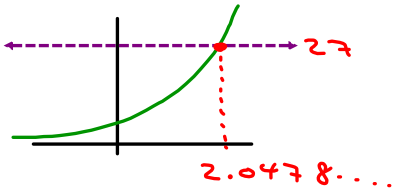
try 2.05 ; $5^{2.05} = 27.1$

→ slow and lame!

② graph $y = 5^x$

$y = 27$

get intersection:



③ try this

$\log_5 27 = \frac{\log 27}{\log 5} = 2.0478...$

Change of base formula: $\log_b(a) = \frac{\log_x(a)}{\log_x(b)}$

may take on ANY legit. value !!! but your calculator uses base 10

or try $\ln = \log_e$

natural logarithm

Practice: Estimate first, and then calculate.

(a) $\log_2(32.5)$

$2^{5.0} = 32.5$

estimate 5.05

$\frac{\log 32.5}{\log 2} \doteq 5.02$

(b) $\log_3(100)$

$3^4 = 81$

$3^5 = 243$

estimate 4.1

$\frac{\ln 100}{\ln 3} \doteq 4.19$

(c) $\log_7(1000)$

$7^3 = 343$

$7^4 = 2401$

1000

estimate 3.4

$\frac{\ln 1000}{\ln 7} \doteq 3.55$

General Properties of Logarithms

GP#1: $\log_b(1) = 0$

think exponentially

$\boxed{b^0 = 1}$

Proof:

let $y = \log_b 1$

$b^y = 1$

$b^y = b^0$

$\therefore \boxed{y = 0}$

GP#2: $\log_b(b^x) = x$

$b^x = b^x$

duh...

let $y = \log_b b^x$

$b^y = b^x$

$\therefore y = x$

$\therefore \log_b b^x = x$

GP#3: $b^{\underline{\log_b(x)}} = x$ ← this in exponential form

$\log_b x = \log_b x$ ← convert to log form

∴ the exponent is the answer

Applications of Logarithms

The half-life of a substance is determined by the formula $$M(t) = P\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

where M(t) is the current amount, P is the original amount and h is the half-life

Assume the half-life of a new substance called Mathium is 40 days.

(a) If there are 20kg of Mathium originally, how many will there be after 100 days?

$$M(100) = 20\left(\frac{1}{2}\right)^{100/40}$$

$$= 20\left(\frac{1}{2}\right)^{2.5}$$

$$= 3.54 \text{ kg}$$

(b) How long will it take for there to be only 2.5 kg of Mathium?

$$\frac{2.5}{20} = \frac{20\left(\frac{1}{2}\right)^{t/40}}{20}$$

$$\left(\frac{1}{8}\right) = \left(\frac{1}{2}\right)^{t/40}$$

$$\left(\frac{1}{2}\right)^{3} = \left(\frac{1}{2}\right)^{t/40} \rightarrow 3 = \frac{t}{40}$$

$$\boxed{t = 120 \text{ days}}$$

(c) How long will it take for there to be only 2 kg of Mathium?

$$\frac{2}{20} = \frac{20\left(\frac{1}{2}\right)^{t/40}}{20}$$

$$0.1 = \left(\frac{1}{2}\right)^{t/40}$$

convert to log form → $\log_{\frac{1}{2}} 0.1 = \frac{t}{40}$

use base change!

$$\frac{\log 0.1}{\log 1/2} = \frac{t}{40}$$

$$3.322 = \frac{t}{40}$$

$$\boxed{t \doteq 132.9 \text{ days}}$$

Homefun... I know it will be.

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