8.4 Graphs of Sinusoidal Functions

Desmos: Trigonometric Graphing & Graphing the Sine Function

EXAMPLE 1 Determining the characteristics of a cosine function based on its equation

Consider the function

$$y = 2 \cos 4x + 1$$

for $\{x \mid 0° \leq x \leq 360°, x \in \mathrm{R}\}$.

$b = 4$ the # of cycles in 2π

$T = \frac{2\pi}{b}$

$T = \frac{2\pi}{4}$

$\boxed{T = \frac{\pi}{2}}$

a) Describe the graph of the function by stating the amplitude, equation of the midline, range, and period, as well as the relevant horizontal translation of $y = \cos x$. $y = 1$

Range: $\{y \in \mathrm{R} \mid -1 \leq y \leq 3\}$

b) Verify your description by drawing a graph of this function using graphing technology.

Repeat for: $y = 5 \cos \frac{1}{2}x - 3$

Range: $\{y \in \mathrm{R} \mid -8 \leq y \leq 2\}$

amp = 5 midline: $y = -3$ $T = \frac{2\pi}{1/2} = 4\pi$

EXAMPLE 2 Determining the characteristics of a sine function based on its equation

Consider the function

$$y = 3 \sin 2(x - 45°)$$

b

45° to the right

a) Describe the graph of the function by stating the amplitude, equation of the midline, range, and period, as well as the relevant horizontal translation of $y = \sin x$.

phase shift

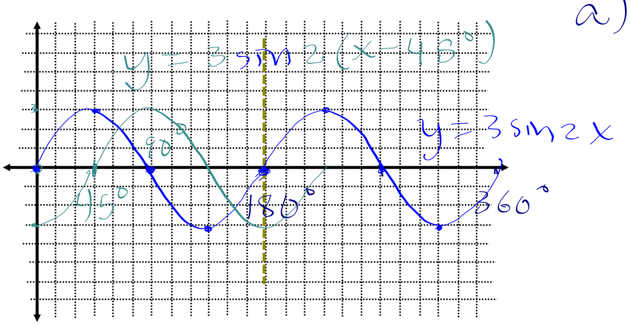
b) Verify your description by drawing the graph using graphing technology.

c) How would the graph of $y = 3 \cos 2(x + 45°)$ be the same? How would it be different?

a) amp = 3

$T = \frac{2\pi}{2} = \pi = 180°$

phase shift of 45°

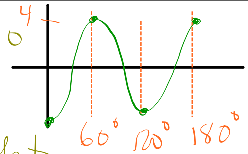


or $y = -3\cos 2x$

Without graphing first, repeat for: $y = 4\cos 3(x - 60^\circ) + 0$

midline : $y = 0$ period : $T = \frac{2\pi}{3} = 120^\circ$

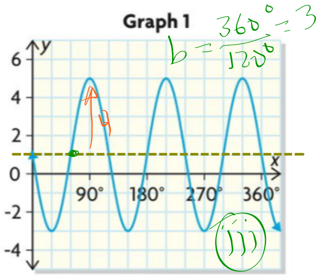
range: $-4 \le y \le 4$ phase shift : 60° right

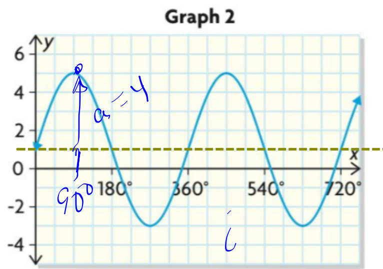


$\boxed{y = -4\cos 3x}$

EXAMPLE 3 Matching equations to graphs

Match each graph with the corresponding equation below.

a) 

b) 

$b = \frac{360^\circ}{360^\circ}$

$b = 1$

i) $y = 4\cos(x - 90^\circ) + 1$

ii) $y = 5\sin 3(x - 60^\circ)$

iii) $y = 4\sin 3(x - 60^\circ) + 1$

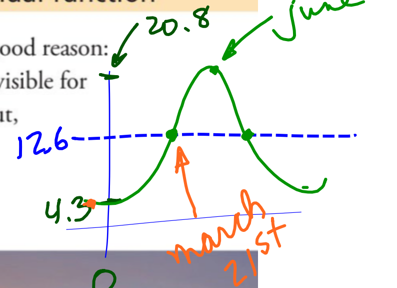
iv) $y = 4\cos 3(x - 60^\circ) + 1$

EXAMPLE 4 Solving a problem using a sinusoidal function

The Far North is called "the Land of the Midnight Sun" for a good reason: during the summer months, in some locations, the Sun can be visible for 24 h a day. The number of hours of daylight in Iqaluit, Nunavut, can be represented by the function

$$y = 8.245\sin\left[0.0172(x - 80.988)\right] + 12.585$$

where x is the day number in the year.



a) How many hours of daylight occur in Iqaluit on the following days?
   i) the shortest day of the year ≈ 4.3
   ii) the longest day of the year ≈ 20.8

b) In some years, June 21 is the longest day. Suppose that the Sun were to set in Iqaluit at 11:01 p.m. on June 21. At what time did the Sun rise? → 3.2 hrs of no Sun → 2:13 am

c) What is the period of this sinusoidal function? Explain how the period relates to the context of the problem.

d) What does the value of c, 80.988, represent in the context of the problem?

The Sun rising in Iqaluit on the shortest day

Pg. 558 # 5-9, 11, 13, 15, 16, 21

c) $T = \frac{2\pi}{b}$

$= \frac{2\pi}{0.0172}$

$= 365.3$ days

d) phase shift = 81 days to the right

81 days to the first equinox (march 21) which is where the sin graph begins

In Summary

Key Idea

- Any sinusoidal function can be expressed as either a cosine function or a sine function.

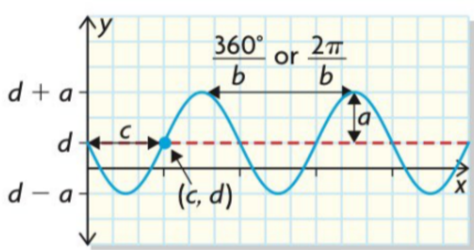
Need to Know

phase shift of c units to the right

- A sinusoidal function of the form

$$y = a \sin b(x - c) + d \text{ or}$$
$$y = a \cos b(x - c) + d$$

has the following characteristics:
  - The value of a is the **amplitude:**
  $$a = \frac{\text{maximum value} - \text{minimum value}}{2}$$
  - The value of b is the number of cycles in 360° or 2π. The **period** is $\frac{360°}{b}$ or $\frac{2\pi}{b}$.
  - The value of c indicates the horizontal translation (phase shift) that has been applied to the graph of $y = \sin x$ or $y = \cos x$.
  - The **equation of the midline** is
  $$y = d$$
  where
  $$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$
  - The **maximum value** is $d + a$, and the **minimum value** is $d - a$.
- In the graph of a sine function, c is the distance from the vertical axis to the first midline point where the function is increasing.



- In the graph of a cosine function, c is the distance from the vertical axis to the first maximum point.