8.4 Logarithm Laws

Recall exponent rules:

Product rule: $a^x \cdot a^y = a^{x+y} \Rightarrow x^9 \cdot x^3 = x^{12}$

Quotient rule: $\frac{a^x}{a^y} = a^{x-y} \Rightarrow \frac{x^9}{x^3} = x^6$

Power rule: $(a^x)^y = a^{xy} \Rightarrow (z^2)^5 = z^{10}$

Remember that logarithms are exponents so the logarithm laws follow the pattern of the EXPONENTS in the exponent rules. → as with exponent laws, log laws only work when the bases are the same!

PRODUCT LAW OF LOGARITHMS

Let $\mathbf{m = a^x}$ and $\mathbf{n = a^y}$

Then... $mn = a^x a^y$

$mn = a^{x+y}$

$\log_a mn = x+y$ ⇐ into log form

but $m = a^x \Rightarrow \log_a m = x$

and $n = a^y \Rightarrow \log_a n = y$ } sub into

$$\log_a mn = \log_a m + \log_a n$$

ex// $\log_3 (27)(3) = \log_3 27 + \log_3 3$

$= 3 + 1$

$= 4$

QUOTIENT LAW OF LOGARITHMS

Let $m = a^x$ and $n = a^y$ $\Rightarrow$ $\log_a m = x$ and $\log_a n = y$

Then...

$$\frac{m}{n} = \frac{a^x}{a^y}$$

$$\frac{m}{n} = a^{x-y}$$

log form $\Rightarrow$ $\log_a\left(\frac{m}{n}\right) = x - y$

$$\boxed{\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n}$$

POWER LAW OF LOGARITHMS

Let $m = a^x$

Then...

$$(m)^d = (a^x)^d$$

$$m^d = a^{xd}$$

log form $\Rightarrow$ $\log_a(m^d) = xd$

but $\log_a m = x$

$$\log_a(m^d) = (\log_a m)\,d$$

$$\boxed{\log_a m^d = d \log_a m}$$

ex// $\log_3 3^5$

$= 5\log_3 3$

$= 5(1)$

$= 5$

Simplify each logarithm expression:

(a) $\log_3 6 + \log_3 4.5$

$= \log_3 (6)(4.5)$

$= \log_3 27$

$= 3$

(b) $\log_2 48 - \log_2 3$

$= \log_2 \left(\frac{48}{3}\right)$

$= \log_2 16$

$= 4$

(c) $\log_5 \sqrt[3]{25}$

$= \log_5 (25)^{1/3}$

$= \frac{1}{3} \log_5 25$

$= \frac{1}{3}(2)$

$= \frac{2}{3}$

Example: If $\log_a x = 2$, $\log_a y = 5$ and $\log_a z = -1$ use the laws of logarithms to calculate the value of...

$$\log_a \sqrt{\frac{x^3 y^2}{z}}$$

$= \log_a \left(\frac{x^3 y^2}{z}\right)^{1/2}$

$= \frac{1}{2} \log_a \left(\frac{x^3 y^2}{z}\right)$

$= \frac{1}{2} \left[\log_a x^3 y^2 - \log_a z\right]$

$= \frac{1}{2} \left[\log_a x^3 + \log_a y^2 - \log_a z\right]$

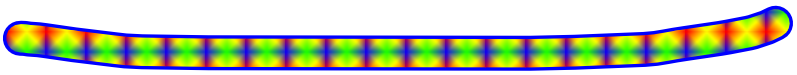
$= \frac{1}{2} \left[3 \log_a x + 2 \log_a y - \log_a z\right]$

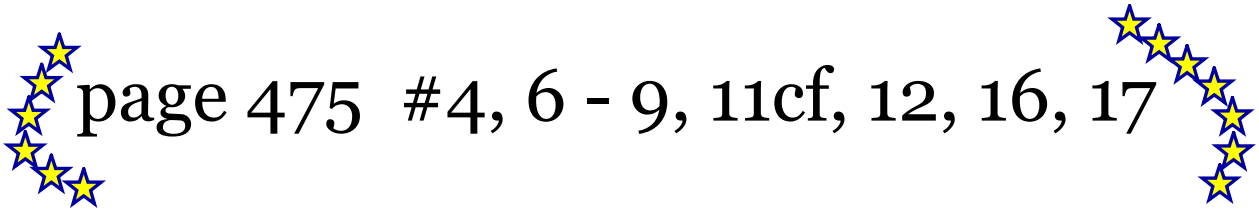
$= \frac{1}{2} \left[3(2) + 2(5) - (-1)\right]$

$= \frac{1}{2}(17)$

$= \frac{17}{2}$

Homefun:





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