8.5 Solving Exponential Equations

An exponential equation is... when the variable is in the exponent: ex// $3^{x+1}=27$

Strategy 1: Write both sides with the same base.

Solve: $\frac{1}{16} = 2^{2x-6}$

express as base 2

$2^{-4} = 2^{2x-6}$

∴ since bases are the same, the exponents are equal

$-4 = 2x-6$

$\frac{2}{2} = \frac{2x}{2}$

$\boxed{x=1}$

Strategy 2: Use logarithms.

Solve for the value of y. $486 = 2(3)^{7-y}$ Three "looks" at the problem...

① write LS as base 3

$\frac{486}{2} = \frac{2(3)^{7-y}}{2}$

$243 = 3^{7-y}$

$3^5 = 3^{7-y}$

$5 = 7-y$

$\boxed{y=2}$

② use the log form

$243 = 3^{7-y}$

$\log_3 243 = 7-y$

base change

$\frac{\log 243}{\log 3} = 7-y$

$y = 7 - \frac{\log 243}{\log 3}$

$\boxed{y=2}$

③ use logarithms

$243 = 3^{7-y}$

log both sides

$\log 243 = \log 3^{(7-y)}$

$\frac{\log 243}{\log 3} = \frac{(7-y)\log 3}{\log 3}$

$\frac{\log 243}{\log 3} = 7-y$

$y = 7 - \frac{\log 243}{\log 3}$

$\boxed{y=2}$

Example 1:

Solve $49^{x-1} = 7\sqrt{7}$

$(7^2)^{x-1} = 7^1(7^{1/2})$

$7^{2x-2} = 7^{2/2 + 1/2}$

$2x - 2 = 3/2$

$2x = \frac{4}{2} + \frac{3}{2}$

$2x = \frac{7}{2}$

$\boxed{x = \frac{7}{4}}$

or log. both sides:

$\log 49^{x-1} = \log 7\sqrt{7}$

$\frac{(x-1)\log 49}{\log 49} = \frac{\log 7\sqrt{7}}{\log 49}$

$x - 1 = \frac{\log 7\sqrt{7}}{\log 49}$

$x = \frac{\log 7\sqrt{7}}{\log 49} + 1$

$\boxed{x = 1.75}$

Example 2:

Solve $2^{x+2} - 2^x = 24$

$2^x \cdot 2^2 - 2^x = 24$

$2^x(2^2 - 1) = 24$

$2^x(4 - 1) = 24$

$\frac{2^x(3)}{3} = \frac{24}{3}$

$2^x = 8$

$\boxed{x = 3}$

Example 3: A problem from last year you couldn't solve... now you can!

You invest $4500 in a Canadian bank that is giving you 6% interest, compounded monthly. How long will it take for the money to grow to $6000? Recall the compound interest formula is $A = P(1 + i)^n$

$A = P(1+i)^n$

$A = 6000$
$P = 4500$
$i = \frac{0.06}{12} = 0.005$
$n = ?$

$$\frac{6000}{4500} = \frac{4500(1.005)^n}{4500}$$

$$1.\overline{3} = 1.005^n$$

how about $\underline{\ln}$ing both sides? (could use logs)

$$\ln\left(\frac{4}{3}\right) = \ln 1.005^n$$

$$\frac{\ln \frac{4}{3}}{\ln 1.005} = \frac{n \ln 1.005}{\ln 1.005}$$

$$\boxed{n \doteq 57.7}$$

$\therefore$ It would take 58 months.

Example 4: Solve for x to 2 decimal places. $2^{x+1} = 3^{x-1}$

$\log 2^{x+1} = \log 3^{x-1}$

$(x+1)\log 2 = (x-1)\log 3$ distribute

$x\log 2 + \log 2 = x\log 3 - \log 3$ collect like terms

$x\log 2 - x\log 3 = -\log 2 - \log 3$ factor both sides

$$\frac{x(\log 2 - \log 3)}{\log 2 - \log 3} = \frac{-(\log 2 + \log 3)}{(\log 2 - \log 3)}$$

$$x = \frac{\log 2 + \log 3}{\log 3 - \log 2}$$

$$x = \frac{\log 6}{\log 3/2}$$

$$x \doteq 4.42$$

Homefun:

page 485 #3ace, 4, 5cdef, 6, 8ace, 10c, 14