8.5 Modelling Data with Sinusoidal Functions

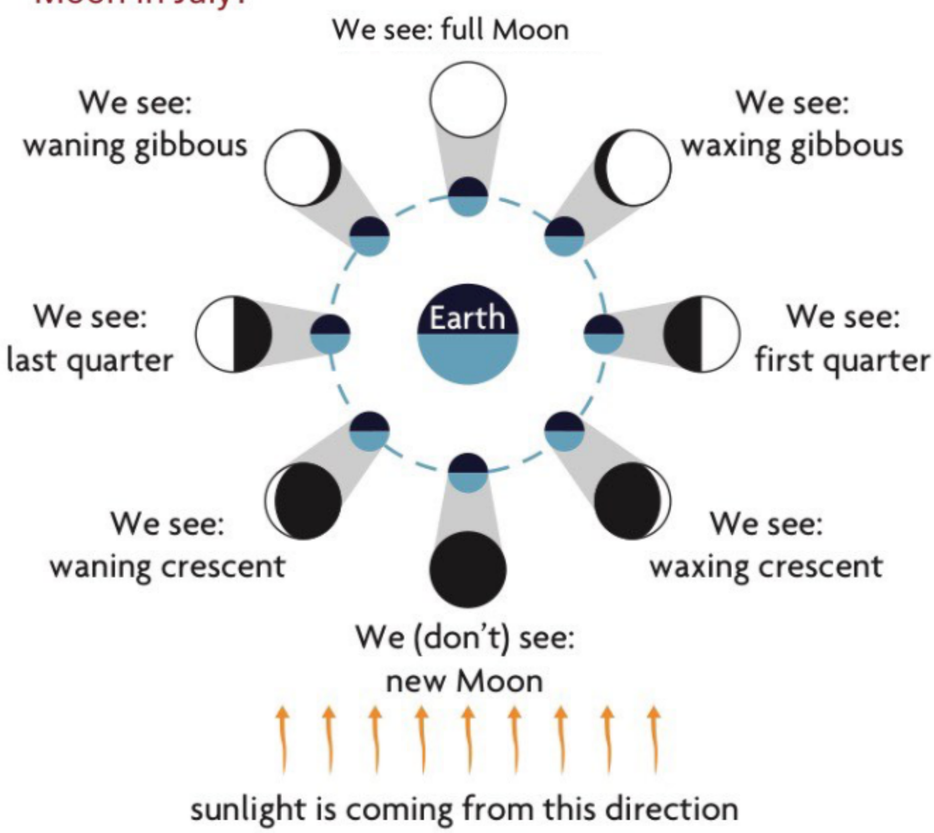
Kelly lives in Winnipeg, Manitoba. She walks her dog at the same time each evening. She noticed that a different percent of the Moon is illuminated each evening, so she decided to look for patterns. She recorded the following data for three months, beginning on April 1.

Percent of the Moon Visible Each Evening

Day	1	3	5	7	9	11	13	15	17	19	21	23
Percent (%)	2	0	6	20	41	64	85	96	100	95	82	63
Day	25	27	29	31	33	35	37	39	41	43	45	47
Percent (%)	43	25	10	1	0	2	12	30	52	75	93	98
Day	49	51	53	55	57	59	61	63	65	67	69	71
Percent (%)	99	89	72	53	33	17	5	0	4	17	38	61
Day	73	75	77	79	81	83	85	87	89	91		
Percent (%)	83	97	100	97	86	69	50	31	14	3		

92 = July 1

Kelly is planning a camping trip in July, and she wants her trip to occur during a full Moon. When can she expect to see a full Moon in July?



C. The synodic period is the length of time between full Moons. Estimate the synodic period using your graph. max to max ≈ 30

D. Enter the data into your graphing calculator. Create a scatter plot.

E. Determine the equation of the sinusoidal regression function that models the data. $y = 50.5 \sin(0.21x - 2.04) + 48.92$

F. Comment on the accuracy of your regression equation. $T = \frac{2\pi}{0.21} = 30.2$

G. Determine the synodic period using your regression equation.

H. On what date in July will the full Moon occur? Explain.

day 77 + 1 period = day 107.9
= July 17th

Reflecting

I. The synodic period of the Moon is actually 29.53 days. Compare your result with this value. close

J. Would you rely on a prediction made from Kelly's data? Explain. yes

important data are the highs & lows

K. To save time, Betty entered only every fourth data point in her graphing calculator. Would she get the same sinusoidal regression equation that you did? Would Betty's prediction be as reliable? Explain.

yes

EXAMPLE 1 Solving an interpolation problem using a sinusoidal model

Celeste lives in Red Deer, Alberta. The predicted hours of daylight for two consecutive years are shown in the tables below. In the second year, the spring equinox will occur on March 20 and the fall equinox will occur on September 23. Compare the hours of daylight on these two days.

Hours of Daylight in Red Deer This Year		
Date	**Day Number**	**Length of Day (h)**
Jan. 1	1	7.812
Feb. 1	32	9.113
Mar. 1	60	10.896
Apr. 1	91	12.998
May 1	121	14.944
Jun. 1	152	16.455
Jul. 1	182	16.690
Aug. 1	213	15.494
Sep. 1	244	13.595
Oct. 1	274	11.597
Nov. 1	305	9.580
Dec. 1	335	8.064

Predicted Hours of Daylight in Red Deer Next Year		
Date	**Day Number**	**Length of Day (h)**
Jan. 1	366	7.808
Feb. 1	397	9.100
Mar. 1	425	10.880
Apr. 1	456	12.982
May 1	486	14.929
Jun. 1	517	16.447
Jul. 1	547	16.694
Aug. 1	578	15.507
Sep. 1	609	13.611
Oct. 1	639	11.613
Nov. 1	670	9.595
Dec. 1	700	8.073
Jan. 1	731	7.803

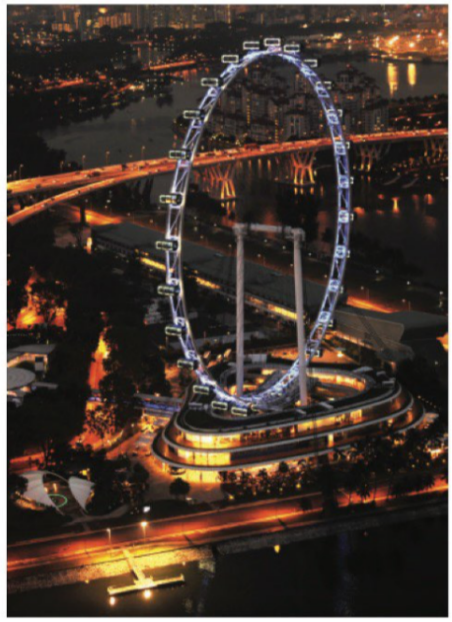
Your Turn

Suppose that the summer solstice will be on June 21 and the winter solstice will be on December 21 this year. Determine the hours of daylight in Red Deer on each day. Round to two decimal places.

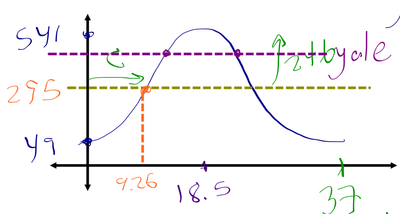
EXAMPLE 3 Solving a problem using a sinusoidal model

In 2011, the Singapore Flyer was the largest Ferris wheel in the world. The table below gives the height of a rider from the ground at different times.

Time (min)	Height (ft)
0	49
9.25	295
18.50	541
27.75	295
37.00	49
46.25	295
55.50	541
64.75	295
74.00	49



Jordy got on the Singapore Flyer at noon and rode it for four consecutive rotations. His friend Yale was in a building directly across from the Singapore Flyer, at a height of 400 ft. When was Jordy level with Yale?



regression:

$y = 246\sin(0.1698x - 1.5708) + 295$

observation: $b = \frac{2\pi}{T} = \frac{2\pi}{37}$

without regression

$y = a\sin[b(x-c)] + d$

$y_2 = 400$ & get POI.

$t_1 = 11.84$ $t_2 = 25.15$

$\boxed{y = 246\sin[0.1698(x-9.25)] + 295}$

Your Turn

At what time was Jordy at a height of 500 ft for the fifth time?

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↳ extend window to see at least 2.5 cycles and find POI ⇒ $t \doteq 89$