Applications of Logarithms

There are many applications of logarithms that are used. Three examples of common use would be: pH scale, Richter scale and the deciBel scale.

We use logarithmic scales in order to compare values that are many orders of magnitude in difference from one another.

The Richter Scale

The Richter scale is used to measure the comparitive magnitude of earthquakes.

The magnitude M of an earthquake is measured by the equation:

$M = \log[I/I_0]$ where I_0 is the intensity of a baseline earthquake.

exponential form $10^m = \frac{I}{I_0}$

(a) write the equation in exponential form.

(b) what is the magnitude of a quake which is 160 000 times more intense than the baseline quake?

(c) If the magnitude of Earthquake 1 is measured at 4.3 on the Richter scale, what will be the magnitude of Earthquake 2 if it is 1500 times more intense than earthquake 1. (You may need to use 2 equations for this one)

b) new quake

$10^m = \frac{I}{I_0}$

$10^m = \frac{160\,000\, I_0}{I_0}$

into log form

$\log 160000 = m$

$m \doteq 5.2$

c) * Intensity I'm looking for $= 10^x$

* intensity I'm comparing with is $10^{4.3}$

$\therefore \frac{10^x}{10^{4.3}} = 1500$

$10^{x-4.3} = 1500$ log form

$\log 1500 = x - 4.3$

$3.176 \doteq x - 4.3$

$x \doteq 3.176 + 4.3$

$x \doteq 7.476$

The pH Scale

Concentration of hydrogen ions compared to distilled water		Examples of solutions at this pH
10 000 000	pH = 0	battery acid, strong hydrofluoric acid
1 000 000	pH = 1	hydrochloric acid secreted by stomach lining
100 000	pH = 2	lemon juice, gastric acid, vinegar
10 000	pH = 3	grapefruit, orange juice, soda
1000	pH = 4	tomato juice, acid rain
100	pH = 5	soft drinking water, black coffee
10	pH = 6	urine, saliva
1	pH = 7	"pure" water
$\frac{1}{10}$	pH = 8	seawater
$\frac{1}{100}$	pH = 9	baking soda
$\frac{1}{1000}$	pH = 10	Great Salt Lake, milk of magnesia
$\frac{1}{10\,000}$	pH = 11	ammonia solution
$\frac{1}{100\,000}$	pH = 12	soapy water
$\frac{1}{1\,000\,000}$	pH = 13	bleaches, oven cleaner
$\frac{1}{10\,000\,000}$	pH = 14	liquid drain cleaner

A difference of one pH unit represents a tenfold (10 times) change in the concentration of hydrogen ions in the solution. For example, the acidity of a sample with a pH of 5 is 10 times greater than the acidity of a sample with a pH of 6. A difference of 2 units, from 6 to 4, would mean that the acidity is 100 times greater, and so on.

- A liquid with a pH less than 7 is considered *acidic.*
- A liquid with a pH greater than 7 is considered *alkaline.*
- A liquid with a pH of 7 is considered *neutral.* Pure distilled water has a pH value of 7.

The relationship between pH and hydrogen ion concentration is given by the formula $pH = -\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions in moles per litre (mol/L).

a) Calculate the pH if the concentration of hydrogen ions is 0.0001 mol/L.

b) The pH of lemon juice is 2. Calculate the hydrogen ion concentration.

c) If the hydrogen ion concentration is a measure of the strength of an acid, how much stronger is an acid with pH 1.6 than an acid with pH 2.5?

a) $pH = -\log[0.0001]$

$= 4$

b) $-2 = -\log[H^+]$

$10^{-2} = H^+$

$H^+ = 0.01$ Mol/L

c) $\frac{10^{2.5}}{10^{1.6}}$

$= 10^{2.5-1.6}$

$= 10^{0.9}$

$= 7.9$

∴ pH is 7.9 times more acidic.

The deciBel Scale

The deciBel scale measures the loudness, L, of a sound. It compares the sound to the quietest sound able to be heard by a human. This sound, measured at 0.000 000 000 001 W/m² is given a loudness of 0dB. In comparison, louder sounds that are encounted may be many million or billion times this level (which is an impractical scale to use). This is the main reason for the deciBel scale.

∴ In short, divide dBels by 10 and treat as pH or Richter

* think Bels ⇒ Richter ⇒ pH but here

Sound	Loudness (dB)
soft whisper	30
normal conversation	60
shouting	80
subway	90
screaming	100
rock concert	120
jet engine	140
space-shuttle launch	180

The loudness of a sound, L, is determined by the equation $L = 10\log[I/I_0]$,

where I is the intensity of a sound and I_0 is the intensity of the baseline sound.

Using the table above and the equation:

(a) What is the loudness of a sound 48 times as intense as the baseline sound?

(b) How many times more intense is a rock concert than a normal conversation?

(c) How many time more intense is a shuttle launch than someone screaming?

a) $\frac{10^x}{10^0} = 48$

$\log 48 = x$

$x \doteq 1.68$ Bels

or $x \doteq 16.8$ dB

b) concert = 120dB = 12B
conversation = 60dB = 6B

$\frac{10^{12}}{10^6} = 10^{12-6} = 10^6$

1000 000 times louder

shuttle = 18 B
scream = 10 B

$\frac{10^{18}}{10^{10}} = 10^8$ times louder

Homefun:

page 500 #6, 8, 10, 11, 12, 15

+ p508 #8