Chapter 9: Relationships between Points, Lines and Planes

9.1 Line/Plane and Line/Line Intersections

How can a line and a plane interact?

Three types of line/plane interactions:

- Line on plane
- Parallel but not on
- Zero pts of intersection

Basic solution method:

1. Get line into parametric form.
2. Sub into plane's Cartesian form.
3. Solve for parameter value.
4. This solution will "tell" you which case it is.
Ex 1: Find the point(s) of intersection between
the line $L: (3, 1, 2) + s(1, -4, -8)$ and
the plane $\pi$: $4x + 2y - z - 8 = 0$

1. **Parametric Equations**
   
   \[
   x = 3 + s \\
   y = 1 - 4s \\
   z = 2 - 8s
   \]

2. **Substitute into $\pi$**
   
   \[
   4(x) + 2(y) - z - 8 = 0 \\
   4(3 + s) + 2(1 - 4s) - (2 - 8s) - 8 = 0 \\
   12 + 4s + 2 - 8s - 2 + 8s - 8 = 0 \\
   4s = -4
   \]

3. **Solve for $s$**
   
   \[
   s = -1
   \]

4. **Conclude**
   
   Since $s$ is a single value, we have a single point of intersection.

   \[\text{Substitute } s = -1 \text{ into parametrics}\]

   \[
   x = 3 + (1) \\
   y = 1 - 4(-1) \\
   z = 2 - 8(-1)
   \]

   \[
   \boxed{x = 4} \quad \boxed{y = 5} \quad \boxed{z = 10}
   \]

   \[\text{Thus, } (2, 5, 10) \text{ is the pt of intersection.}\]

   by $L \cap \pi$
9.1 Intersections of Line-Plane and Line-Line

Ex 2: Find the point(s) of intersection between the line \( L: r = (2, 2, 9) + s(1, 2, 8) \) and

the plane \( \pi: 2x - 5y + z - 6 = 0 \)

① \( x = 2 + 8s \)
    \( y = 2 + 2s \)
    \( z = 9 + 8s \)

② Sub s' solve for s

\[ 2(2+8s) - 5(2+2s) + (9+8s) - 6 = 0 \]
\[ 4 + 16s - 10 - 10s + 9 + 8s - 6 = 0 \]

③ \( 8s = 3 \)

④ Since \( 8s = 3 \) is a non-solution, conclude that \( L \parallel \pi \) are parallel but \( \not\perp \) not coincident.

Ex 3: Find the point(s) of intersection between the line \( L: r = (3, -2, 1) + s(14, -5, -3) \) and

the plane \( \pi: x + y + 3z - 4 = 0 \)

① \( x = 3 + 14s \)
    \( y = -2 - 5s \)
    \( z = 1 - 3s \)

② \( (3 + 14s) + (-2 - 5s) + 3(1 - 3s) - 4 = 0 \)
    \( 3 + 14s - 2 - 5s + 3 - 9s - 4 = 0 \)
    \( 0s = 0 \)

③ Since \( 0s = 0 \) has an \( \infty \) # of sol's conclude that \( L \parallel \pi \) and \( L \) lies on \( \pi \).
Line/Line Intersections

how can a line and a line intersect?

Method of solution: Examine direction vectors... if collinear then either parallel or coincident (we know how to do this already)

If not parallel, then solve the parametric equation system.
Example 1: Find pt of intersection of L₁: \( r = (-3, 1, 4) + p(-1, 1, 4) \) and L₂: \( r = (1, 4, 6) + q(-6, -1, 6) \)

1. Examine direction vectors
   \[ \vec{m}_1 = (-1, 1, 4) \quad \vec{m}_2 = (-6, -1, 6) \]
   Not parallel

2. Write both in parametric form
   \[ L_1: \begin{align*} x &= -3 - p \\ y &= 1 + p \\ z &= 4 + 4p \end{align*} \]
   \[ L_2: \begin{align*} x &= 1 - 6q \\ y &= 4 - 9q \\ z &= 6 + 6q \end{align*} \]

3. Use \( x \) \& \( y \) equations to solve for \( p \) \& \( q \), then check using \( z \) equations.

   \[ \begin{align*}
   x: & \quad -3 - p = 1 - 6q & \text{add equations to eliminate } p \\
   y: & \quad 1 + p = 4 - 9q & \text{to eliminate } p \\
   & \quad -2 = 5 - 7q
   \end{align*} \]

   \[ 9 = 7q \]

   Sub back to get \( p \) ... \( 1 + p = 4 - (1) \)

   \[ p = 2 \]

   Check \( p = 2 \) \& \( q = 1 \) in \( z \) equations

   \[ z: \quad 4 + 4p = 6 + 6q \]
   \[ 4 + 4(2) = 6 + 6(1) \] \( \text{Same = good} \)

   \[ 12 = 12 \]

4. Find coordinates of the POI.

   \[ \begin{align*}
   x &= -3 - p & z &= 4 + 4p \\
   x &= -3 - 2 & z &= 4 + 4(2) \\
   x &= -5 & y &= 3
   \end{align*} \]

   POI = \((-5, 3, 12)\)

NB if lines are skew, then 3rd equation will not be equal.
HOMEFUN

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