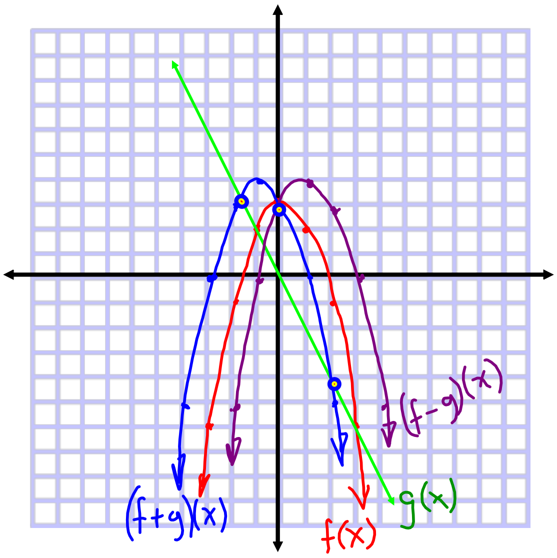
9.2 Combining Functions: Sums and Differences

Two methods: 1. graphing (point by point) 2. algebraic

Example 1: Let $f(x) = -x^2 + 3$ and $g(x) = -2x$. Graph each function and then graph $(f + g)(x)$ and $(f - g)(x)$.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$
-3	-6	6	0	-12
-2	-1	4	3	-5
-1	2	2	4	6
0	3	0	3	3
1	2	-2	0	4
2	-1	-4	-5	3
3	-6	-6	-12	0



algebraically: $(f+g)(x) = f(x) + g(x)$

$= (-x^2+3) + (-2x)$

$= -x^2 - 2x + 3$

$= (x+3)(x-1)$

parabola... opens downs...
roots @ $x = -3$ and $x = 1$

for $(f + g)(x)$ ... zeroes occur when... $f(x) = -g(x)$

y-int... y-int of $f(x)$ + y-int of $g(x)$

when $f(x) = 0$... $(f+g)(x) = g(x)$

when $g(x) = 0$... $(f+g)(x) = f(x)$

what about the degree of the function?

quadratic + linear = quadratic

for $(f - g)(x)$ ... zeroes occur when... $f(x) = g(x)$ ← point of intersection

y-int... y-int of $f(x)$ − y-int of $g(x)$

when $f(x) = 0$... $(f-g)(x) = -g(x)$

when $g(x) = 0$... $(f-g)(x) = f(x)$

what about the degree of the function?

quad − linear = quadratic

A few notes:

① $(f+g)(x) = (g+f)(x)$

② $(f-g)(x) \neq (g-f)(x)$

actually... $= -(g-f)(x)$

③ degree (for polynomials only)

→ if degrees are different, the combined function has the larger degree

ex// $(x^3) + (2x^2-1) = x^3 + 2x^2 - 1$

→ if degrees are =, the combined function has the original degree or lower

ex// $(x^2-2x) - (x^2+3x+1)$

$= -5x - 1$

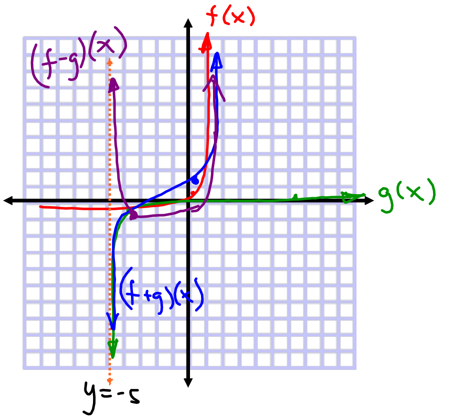
Example 2: What about DOMAIN?

$x+5>0$

$x>-5$

Sketch a graph of (f - g)(x) and (f + g)(x), if f(x) = 10^x and g(x) = log(x + 5) and determine the domain and range of all the functions.

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$
-8	10^{-8}	—	—	—
-6	10^{-6}	—	—	—
-4	10^{-4}	0	10^{-4}	10^{-4}
-2	10^{-2}	0.48	0.49	−0.47
0	1	0.70	1.7	0.3
2	100	0.85	100.85	99.15
4	10000	0.95	big	big
6	10^6	1.04	bigger	bigger



Domain:

* $f(x)$: $x \in (-\infty, \infty)$

* $g(x)$: $x \in (-5, \infty)$

the domain of $(f \pm g)(x)$ is the intersection of the domains of f(x) & g(x)

-5

∴ domain of $(f+g)(x)$: $x \in (-5, \infty)$

ex// State the domain of $(f-g)(x)$ if

$f(x) = \log_2(x+3)$ and $g(x) = \frac{1}{x^2-1} = \frac{1}{(x+1)(x-1)}$

$x \in (-3, \infty)$

$x \in (-\infty,-1) \cup (-1,1) \cup (1,\infty)$

∴ domain for $(f-g)$: $x \in (-3,-1) \cup (-1,1) \cup (1,\infty)$

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