9.5 Compositions of functions

Two functions, *f* and *g*, can combined using a process called composition, which can be represented by *f(g(x))*. The input value for *g* is used as the output value for *f*.

Notation: $f(g(x)) = (f \circ g)(x)$ not multiplication

This is sometimes read as "*f map g*" or *fog(x)*. The point *(a,b)* on the function *g* gets mapped onto the function *f* as (b,c). As a result the point becomes (a,c) or *fog(x)*. See pg. 551.

Ex. For $f(x) = 2x + 3$ *and* $g(x) = \sqrt{x}$, determine:

a) $(f \circ g)(x)$
b) $(g \circ f)(x)$
c) $(f \circ f)(x)$
d) $(g \circ g)(x)$

a) $f(g(x)) = 2\sqrt{x} + 3$

b) $g(f(x)) = \sqrt{2x+3}$

c) $f(f(x)) = 2(2x+3) + 3$
$= 4x + 6 + 3$
$= 4x + 9$

d) $g(g(x)) = \sqrt{\sqrt{x}}$
$= (x^{1/2})^{1/2}$
$= x^{1/4}$ or $\sqrt[4]{x}$

It should be noted ~~that $(f \circ g)(x)$~~ exists ~~only where~~ an element in the range of g is also in the domain of f. The function $(f \circ g)(x)$ exists only when the range of g overlaps the domain of f.

Ex. Let $f(x) = \log_2 x$ and $g(x) = x + 4$

a) determine $(f \circ g)(x)$, and find its domain

$f(g(x)) = \log_2(x+4)$

domain: $x + 4 > 0$

$\boxed{x > -4}$

$\{x \in \mathbb{R} \mid x > -4\}$

or

$x \in (-4, \infty)$

Ex. Given $h(x) = |x^3 - 1|$, find two functions, f and g, such that $h = f \circ g$

$h = f(g(x))$

try $f(x) = |x|$

$g(x) = x^3 - 1$

$\therefore (f \circ g)(x) = |x^3 - 1|$

or

$f(x) = |x - 1|$

$g(x) = x^3$

Ex. If $f(x)=\frac{1}{x-2}$, show that $(f\circ f^{-1})(x)=(f^{-1}\circ f)(x)$.

$f(f^{-1}(x)) = f^{-1}(f(x))$ ... let's find $f^{-1}(x)$

$f(x)=\frac{1}{x-2} \Rightarrow x=\frac{1}{y-2}$

$y-2=\frac{1}{x}$

$y=\frac{1}{x}+2 \Rightarrow \boxed{f^{-1}(x)=\frac{1}{x}+2}$

So ... $f^{-1}(f(x))$

$=\frac{1}{\left(\frac{1}{x-2}\right)}+2$

$=x-2+2$

$=x$

and $f(f^{-1}(x))$

$=\frac{1}{\left(\frac{1}{x}+2\right)-2}$

$=\frac{1}{\frac{1}{x}}$

$=x$

$\boxed{\therefore f\circ f^{-1}=f^{-1}\circ f}$

NB composing a function with its inverse yields $y=x$

Homefun: Pg. 552 #(1-3, 5-7)ac, 11