PRINCIPLES OF MATHEMATICS 12

Trigonometry I Practice Exam

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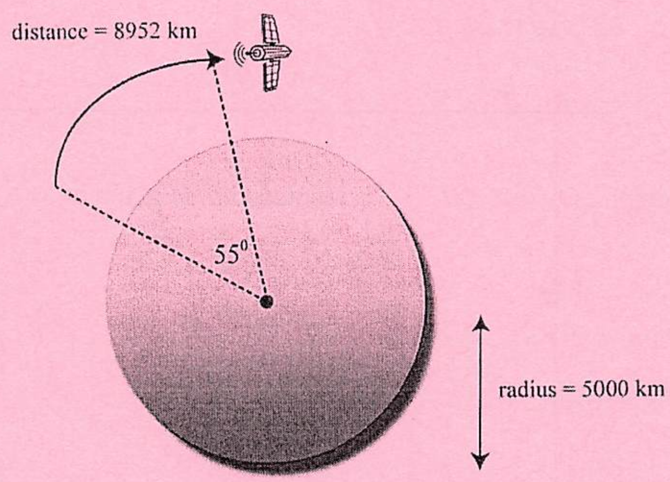
Trigonometry I Practice Exam

1. The transformation $g(\theta) = f(2\theta) - 2$ is applied to the graph of $f(\theta) = \sin\theta$. The range of the new graph is

   A. $-3 \le y \le -1$
   B. $-2 \le y \le 0$
   C. $-3 \le \theta \le -1$
   D. $-2 \le \theta \le 0$

Use the following information to answer the next question.

A satellite is orbiting a small planet, as shown in the following diagram.



2. The height of the satellite above the surface of the planet is, to the nearest km,

   A. 162 km
   B. 3952 km
   C. 4326 km
   D. 5162 km

Numerical Response

1. If the point $\left(\frac{\pi}{2}, -2\right)$ lies on the graph of $f(\theta) = a\cos\left(\theta - \frac{\pi}{4}\right) - 4$, then the value of a, to the nearest tenth, is _________.

Use the following information to answer the next question.

The equation of a trigonometric function is $f(\theta)=k\sin\left(\theta-\frac{\pi}{3}\right)-3,\ k>0$

3. The range of this function is

A. $-3k \le f(\theta) \le 3k$
B. $-k \le f(\theta) \le k$
C. $-3-k \le f(\theta) \le -3+k$
D. $3-k \le f(\theta) \le 3+k$

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4. The graph of $y=\cos\left(\theta+\frac{\pi}{2}\right)$ is identical to the graph of

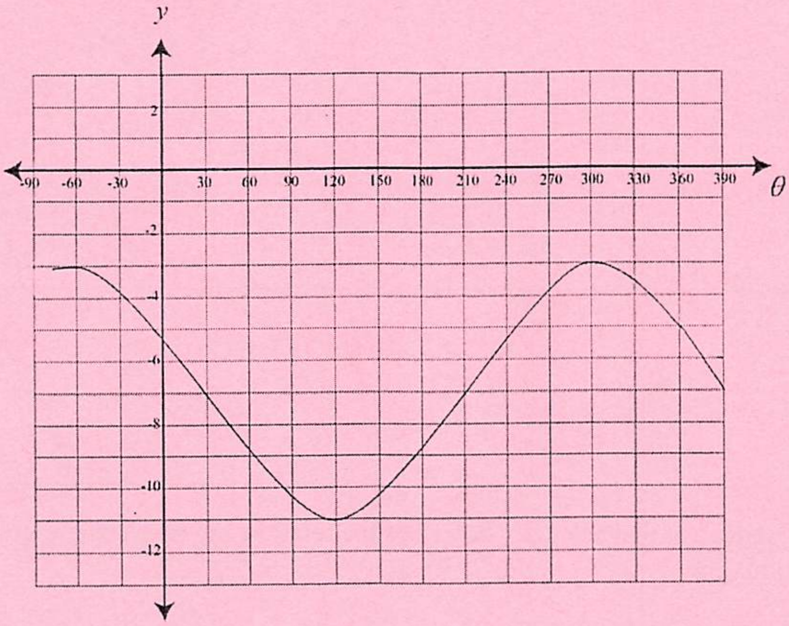
A. $y=-\cos\theta$
B. $y=-\sin\theta$
C. $y=\cos\left(\theta-\frac{\pi}{2}\right)$
D. $y=\sin\theta$

5. The *y*-intercept of the graph represented by $f(\theta)=-3\cos\left(k\theta+\frac{\pi}{2}\right)-b$ is

A. $-b$
B. $3-b$
C. $\frac{3-b}{k}$
D. $\frac{-3-b}{k}$

Use the following information to answer the next two questions.

The partial graph of a trigonometric function is displayed below.



6. An equation that correctly represents this graph is

A. $f(\theta) = -4\sin(\theta - 30^{0}) - 7$

B. $f(\theta) = -4\cos(\theta - 60^{0}) - 7$

C. $f(\theta) = -4\sin(\theta + 60^{0}) - 7$

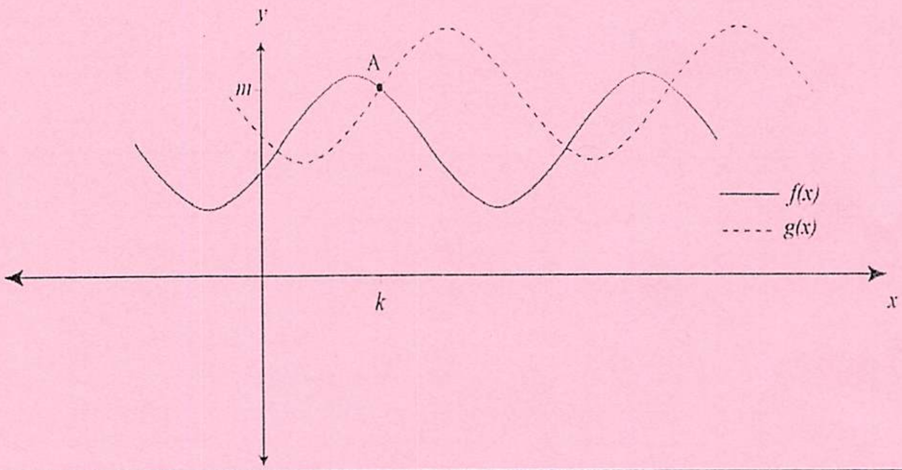
D. $f(\theta) = 4\cos(\theta + 30^{0}) - 7$

7. If the graph above is to be represented by a function in radian mode, rather than degree mode, the parameter(s) which must be changed are

A. a and d
B. b
C. c
D. b and c

Use the following information to answer the next question.

Two trigonometric functions, $f(x)$ and $g(x)$, are graphed below



8. A statement that correctly describes the relationship between the graphs at point A is

A. $f(x) = g(A)$
B. $g(m) = f(m) = k$
C. $f(k) + g(k) = 2m$
D. $g(m) = f(k) = m$

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9. If $\cot\theta = -\frac{3}{4}$ and $\csc\theta < 0$, then the value of $\sin\theta$ is

A. $-\frac{4}{5}$
B. $\frac{4}{5}$
C. $-\frac{3}{5}$
D. $\frac{3}{5}$

10. If $\cos A = \dfrac{\sqrt{3}}{2}$, $0^0 < \theta < 90^0$, and $B = 60^0 + A$, then the value of $\sec B$ is

A. 30^0

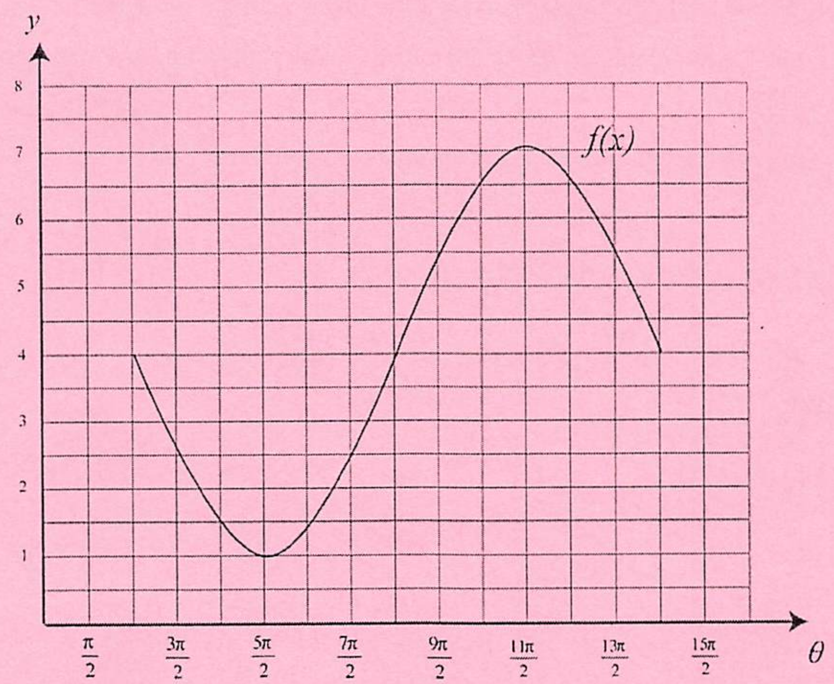
B. $\dfrac{1}{90^0}$

C. 0

D. undefined

Use the following information to answer the next question.

The graph of a trigonometric function *f(x)* is shown below



Numerical Response

2. If the graph above is to be represented in the form $f(\theta) = a\sin\left[b(\theta - c)\right] + d$, then the value of b, to the nearest hundredth, is _________.

Numerical Response

3. If $\cos\theta = -\frac{3}{5}$ and $\tan\theta > 0$, then the value of $\sin^2\theta - \cos^2\theta$ is, to the nearest hundredth, ________.

11. The correct statement regarding the graphs of $f(\theta) = a\sin b\theta$ and $g(\theta) = k\sin\left[b(\theta - c)\right]$ is

A. both graphs have a period equal to b

B. the y-intercept of $g(\theta)$ is a - k units lower than the y-intercept of $f(\theta)$.

C. the θ - intercepts of $g(\theta)$ are c units to the right of the θ - intercepts of $f(\theta)$

D. the y-intercept of $g(\theta)$ is k, and the y-intercept of $f(\theta)$ is a.

12. A graph that has the same y-intercept as $y = \cos\theta$ is

A. $y = 3\cos\theta$

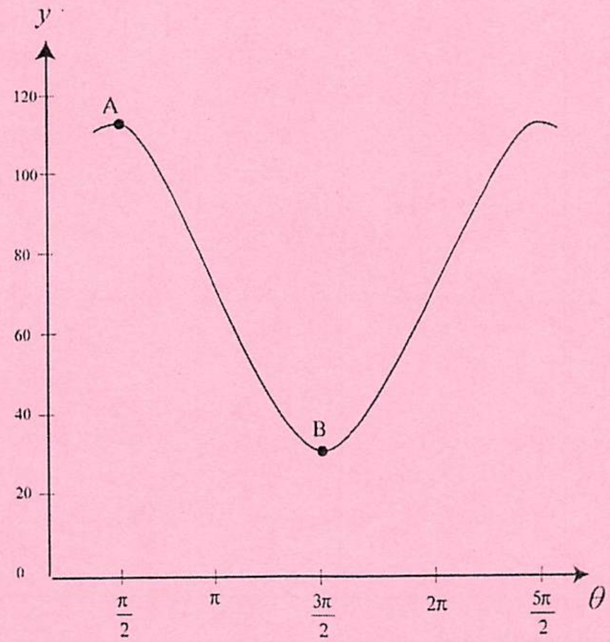
B. $y = \cos 3\theta$

C. $y = \cos(\theta - 3)$

D. $y = \cos\theta + 3$

Use the following information to answer the next question.

The partial graph of a trigonometric function is shown below. The graph has a maximum value $A\left(\frac{\pi}{2}, 112\right)$, and a minimum value $B\left(\frac{3\pi}{2}, 28\right)$



13. An equation that correctly represents the graph shown above is

A. $y = 42\cos\left(\theta - \frac{\pi}{2}\right) + 28$

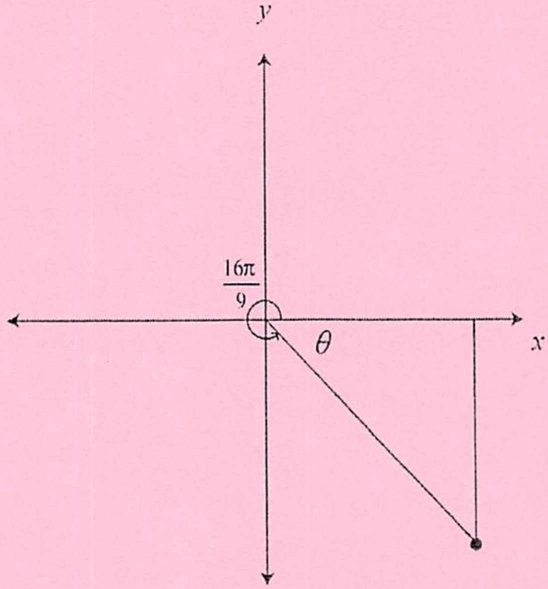
B. $y = 42\cos(\theta - \pi) + 70$

C. $y = 42\cos\left(\theta - \frac{\pi}{2}\right) + 70$

D. $y = 42\cos\left(\theta - \frac{3\pi}{2}\right) + 70$

Use the following information to answer the next question.

A point is on a terminal arm in standard position, as shown below.
The standard angle of the terminal arm is $\frac{16\pi}{9}$



14. The reference angle θ is

A. $\frac{2\pi}{9}$

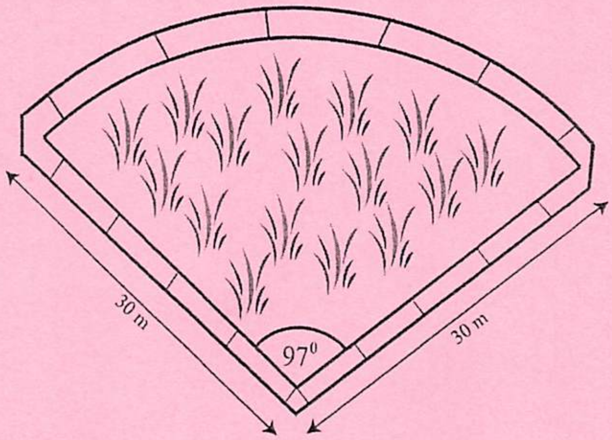
B. 320°

C. $-\frac{63}{16\pi}$

D. $\frac{5\pi}{18}$

Use the following information to answer the next question.

A sidewalk encloses a pie-shaped field, as illustrated below.



Numerical Response

4. The total length of the sidewalk, correct to the nearest metre, is __________.

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15. If $\cos\theta = \frac{4}{5}$, and $\frac{3\pi}{2} < \theta < 2\pi$, the value of $\cot\theta$ is equal to

A. $\frac{3}{5}$

B. $\frac{4}{3}$

C. $-\frac{3}{5}$

D. $-\frac{4}{3}$

16. The graphs of $f(\theta) = \sin 2\theta$ and $g(\theta) = \cos 2\theta$ intersect at the points $\left(\frac{\pi}{8}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{5\pi}{8}, \frac{-\sqrt{2}}{2}\right)$. If the amplitude of each graph is quadrupled, the new points of intersection will be

A. $\left(\frac{\pi}{8}, \frac{\sqrt{2}}{8}\right)$ and $\left(\frac{5\pi}{8}, \frac{-\sqrt{2}}{8}\right)$

B. $\left(\frac{\pi}{8}, \frac{\sqrt{2}}{2}+4\right)$ and $\left(\frac{5\pi}{8}, \frac{-\sqrt{2}}{2}-4\right)$

C. $\left(\frac{\pi}{8}, 2\sqrt{2}\right)$ and $\left(\frac{5\pi}{8}, -2\sqrt{2}\right)$

D. $\left(\frac{\pi}{2}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{5\pi}{2}, \frac{-\sqrt{2}}{2}\right)$

17. The terminal arm of a rotation angle in standard position passes through the point $(8k, -6k)$. If $k > 0$, then the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$ are

A. $-\frac{5}{3}, \frac{5}{4}, -\frac{4}{3}$

B. $-\frac{3}{5}, \frac{4}{5}, -\frac{3}{4}$

C. $\frac{4}{5}, -\frac{3}{4}, -\frac{3}{4}$

D. $-\frac{3}{10}, \frac{7}{10}, -\frac{3}{4}$

18. The exact value of $-3\tan\left(\frac{13\pi}{6}\right)$ is

A. $\sqrt{3}$

B. $-\sqrt{3}$

C. $-\frac{\sqrt{3}}{3}$

D. undefined

Use the following information to answer the next question.

The average wing span of a particular species of butterfly is 8 cm. However, the wing span for new butterflies varies in a periodic manner from year to year. An equation that models the wing span is $w(t) = \cos^3 t - \sin(t-3) + 8$, where $w(t)$ is the wing span in cm, and t is the time in years.

19. A biologist monitors the butterflies over a 25 year period. The range of the wing span is, to the nearest tenth,

A. $0 \le w(t) \le 16.0$

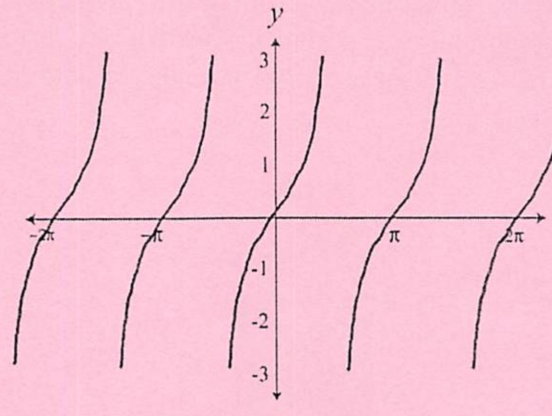
B. $6.7 \le w(t) \le 9.3$

C. $6.8 \le w(t) \le 9.2$

D. $7.0 \le w(t) \le 9.0$

Use the following information to answer the next question.

A student uses technology to draw the graph of $y = \tan\theta$, as shown below.



20. The asymptotes of this graph occur at

A. $\pm n\pi$
B. $\pm 2n\pi$
C. $\frac{\pi}{2} \pm n\frac{\pi}{2}$
D. $\frac{\pi}{2} \pm n\pi$

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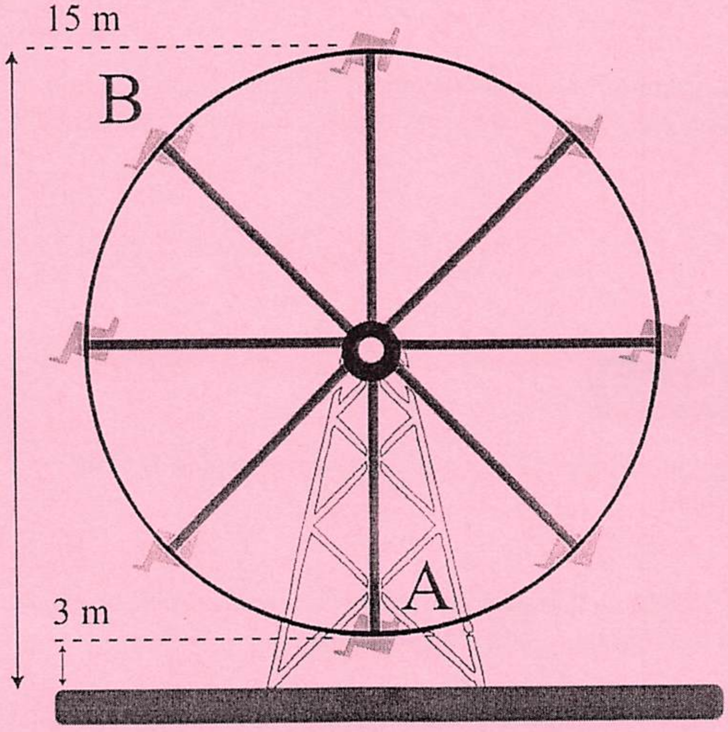
21. All of the following are co-terminal angles to 150° except

A. -930°
B. $\frac{17\pi}{6}$
C. $\frac{23\pi}{6}$
D. -3.67 rad

Use the following information to answer the next ***six*** *questions.*

A Ferris Wheel at an amusement park has riders get on at position A, which is 3 m above the ground. The highest point of the ride is 15 m above the ground. The ride takes 40 seconds for one complete revolution.



22. A function of the form $h(t) = a\cos\left[b(t-c)\right] + d$ can be used to accurately model the height of a Ferris Wheel over time. An equation that correctly models the Ferris Wheel shown above is

A. $h(t) = -6\cos 9t + 9$

B. $h(t) = -6\cos 40\pi t + 9$

C. $h(t) = -6\cos\frac{\pi}{3}t + 9$

D. $h(t) = -6\cos\frac{\pi}{20}t + 9$

23. The time for a rider, who starts at position A, to travel to position B (a rotation of 135°) is

A. 12 s
B. 13 s
C. 14 s
D. 15 s

24. If the ride makes three complete rotations, the total amount of time a rider on the Ferris Wheel will spend above 13 m, rounded to the nearest second, is

A. 11 s
B. 15 s
C. 25 s
D. 32 s

Numerical Response

6. The height of the rider 22 seconds after the ride begins is, to the nearest tenth, ________.

25. If the Ferris Wheel rotates counter-clockwise, instead of the original clockwise motion, the new graph is best represented by

A. changing the sign of the leading coefficient.
B. applying the transformation $y = f(t-40)$
C. applying the transformation $y = f(-t)$
D. using a sine function instead of a cosine function, with no change to the parameters.

26. The ride operator decides to speed up the ride. This will affect parameter

A. a
B. b
C. c
D. d

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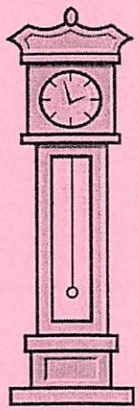
27. If $f(\theta) = \sin 4\theta$, where $0 \le \theta < 3\pi$, then the number of vertical asymptotes in the graph of $\dfrac{1}{f(\theta)}$ is

A. 8
B. 9
C. 12
D. 13

Use the following information to answer the next question.

The pendulum of a grandfather clock swings back and forth with a periodic motion that can be represented by a trigonometric function. At rest, the pendulum is 20 cm above the base . The highest point of the swing is 26 cm above the base, and it takes two seconds for a complete swing back and forth.



28. A cosine equation that models the height of the pendulum as a function of time, if the pendulum is released from the highest point, is

A. $h(t) = 6\cos \pi t + 23$
B. $h(t) = 3\cos \pi t + 20$
C. $h(t) = 3\cos 2\pi t + 20$
D. $h(t) = 3\cos \pi t + 23$

29. The general solution to the equation $2\sin\theta=\sqrt{3}$ is

A. $\theta=\frac{\pi}{6}\pm n\pi,\ \frac{5\pi}{6}\pm n\pi$

B. $\theta=\frac{\pi}{6}\pm 2n\pi,\ \frac{5\pi}{6}\pm 2n\pi$

C. $\theta=\frac{\pi}{3}\pm 2n\pi,\ \frac{4\pi}{3}\pm 2n\pi$

D. $\theta=\frac{\pi}{3}\pm 2n\pi,\ \frac{2\pi}{3}\pm 2n\pi$

30. An appropriate window setting for the graph of $y=20.1\sin\frac{2\pi}{300}(t-265)+6.2$ is

A. x: [0, 17000, 5000], y: [-20, 30, 10]

B. x: [-265, 0, 50], y: [0, 12.4, 1]

C. x: [0, 600, 100], y: [-15, 30, 5]

D. x: [0, 2π, $\frac{\pi}{2}$], y: [-20, 30, 5]

31. The graph of $g(\theta)=\sin[3\theta-\pi]$ is equivalent to the graph of $y=\sin\theta$ after a

A. horizontal shift of π units right, then a horizontal stretch by a factor of $\frac{1}{3}$.

B. horizontal stretch by a factor of $\frac{1}{3}$, then a horizontal shift of π units right.

C. horizontal stretch by a factor of 3, then a horizontal shift of $\frac{\pi}{3}$ units right.

D. horizontal stretch by a factor of $\frac{1}{3}$ then a horizontal shift of $\frac{\pi}{3}$ units right.

32. The domain of $f\left(\theta\right)=\cot 4\theta$ is

A. $x \in R,\ x \neq \pm\dfrac{n\pi}{4}$

B. $x \in R,\ x \neq \pm\dfrac{n\pi}{2}$

C. $x \in R,\ x \neq \pm n\pi$

D. $x \in R$

Use the following information to answer the next question.

The sunrise and sunset times for Yellowknife (*adjusted to remove the effects of daylight savings time*) are given below.

	June 21, 2006	Dec. 21, 2006
Sunrise	2.57 (2:34 AM)	10.18 (10:11 AM)
Sunset	22.75 (10:45 PM)	15.00 (3:00 PM)

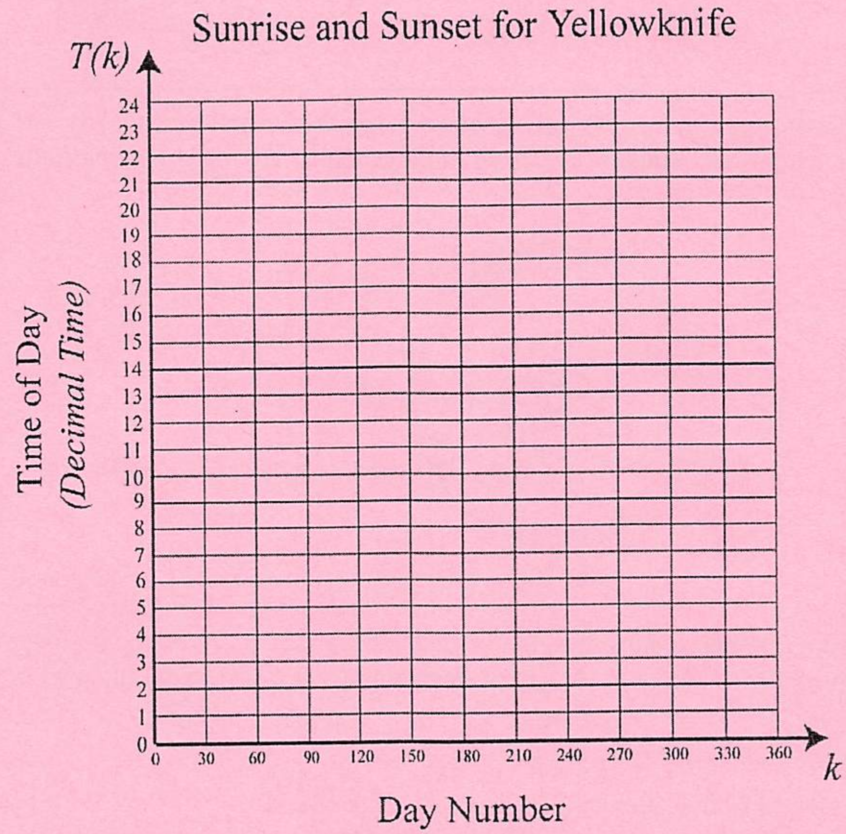
A sinusoidal equation of the form $T(x) = a\cos[b(x-c)] + d$ can be used to graphically model the time of sunrise or sunset throughout the year, where $T(x)$ is the time of day *(using decimal time format)*, and x is the day of the year.

Written Response – 10%

1. • Determine an equation modeling the time of sunrise in Yellowknife.

• Determine an equation modeling the time of sunset in Yellowknife.

- Using technology, graph the functions representing sunrise and sunset times in Yellowknife.



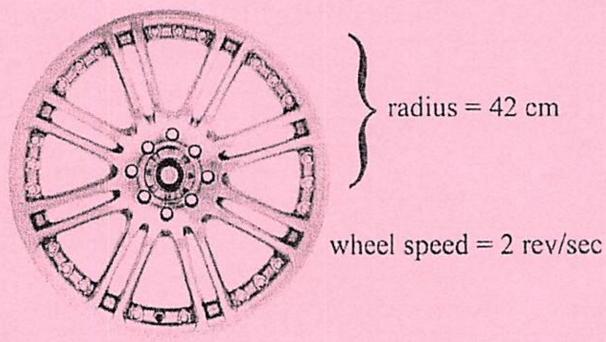
- Mathematically describe the transformations required to change the graph of $f(x) = \cos x$ to the graph representing the sunset time in Yellowknife.

- Determine the number of days Yellowknife experiences a sunrise earlier than 4:00 AM.

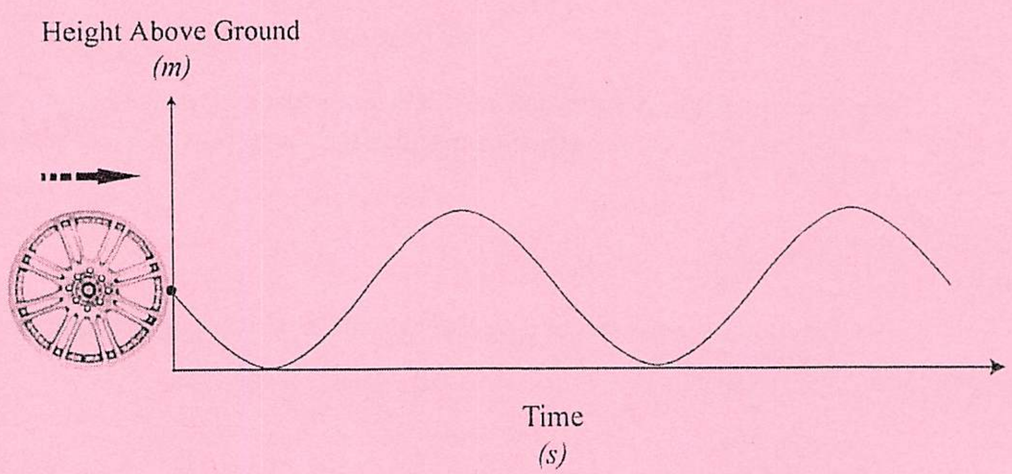
- Determine the number of hours of daylight in Yellowknife on February 15.

Use the following information to answer the next question.

A mechanic changing a tire rolls a wheel along the ground towards the car. The radius of the wheel is 42 cm, and the speed of the wheel as it rolls is 2 revolutions per second.



The diagram below illustrates the vertical motion of a point on the tire over time. It is possible to model the height of this point using a sinusoidal function of the form $h(t) = -a\sin\left[b(t-c)\right] + d$



Written Response – 10%

2.

- Determine the length of time required for one revolution of the tire.

- State the numerical value for each of the parameters a, b, c, & d.

Parameter	Value
a	
b	
c	
d	

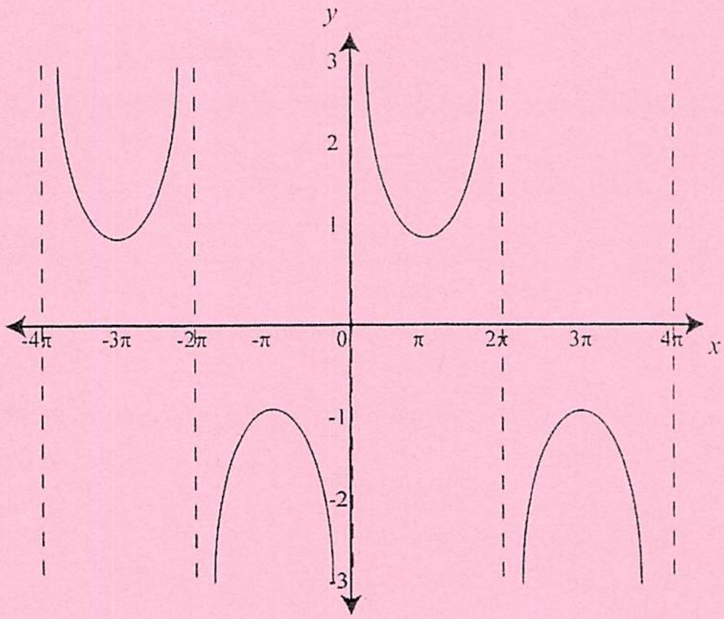
- Write a function representing the motion of the point in the form $h(t) = -a\sin\left[b(t-c)\right] + d$

- Write a formula that predicts the times when contact between the point and ground occur. Use this formula to determine the time when the point touches the ground for the fifth time.

- A second wheel, with a radius of 39 cm, is rolled at the same speed of 2 rev/second. Compare the parameters a, b, c, & d for this wheel with the original wheel.

Use the following information to answer the next question.

The graph of $f(\theta) = \csc \frac{1}{2}\theta$ is shown below:



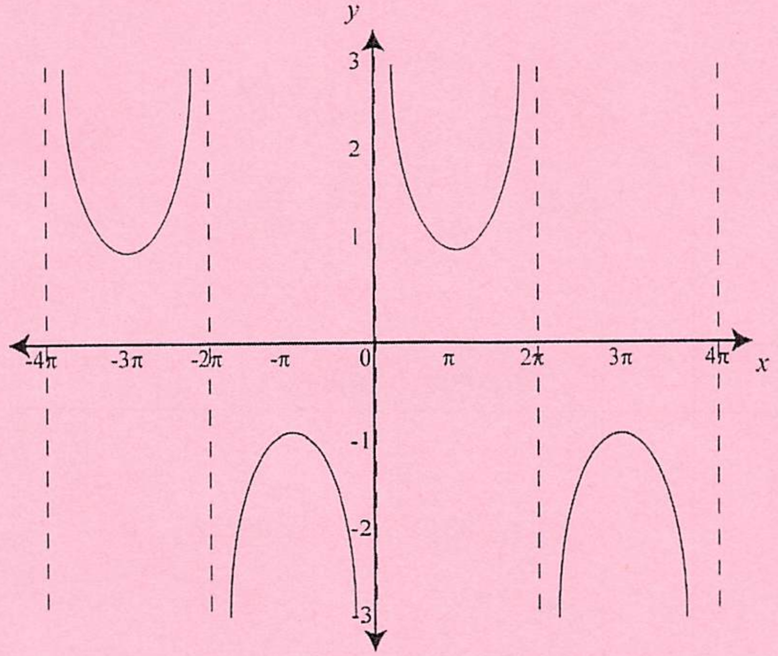
Written Response – 10%

3.

- Complete the following table:

a - value	
b - value	
Phase Shift	
Vertical Displacement	
Period	
Domain	
Range	
x-intercepts	
y-intercepts	
Asymptotes (general equation)	

- Sketch the graph of $\frac{1}{f(\theta)}$ in the space below. Then, write a function $g(\theta)$ that represents the graph you drew in.



- Explain how the location of the asymptotes in $f(\theta)$ can be predicted from the graph of $g(\theta)$.

- Determine the exact value of $f\left(\frac{10\pi}{3}\right)$

You have now completed the examination. Please check over your answers carefully before self-marking. Good luck on your real exam!

Formulas

These are the formulas for Trig I you will be given on your diploma.

$$a = r\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Answer Sheet

1. A	**NR 2)** 0.33	**19.** B	**29.** D
2. C	**NR 3)** 0.28	**20.** D	**30.** C
NR 1) 2.83	**11.** C	**21.** C	**31.** D
3. C	**12.** B	**22.** D	**32.** A
4. B	**13.** C	**23.** D	
5. A	**14.** A	**24.** D	
6. A	**NR 4)** 111	**NR 5)** 14.7	
7. C	**15.** D	**25.** B	
8. C	**16.** C	**26.** B	
9. B	**17.** B	**27.** C	
10. D	**18.** B	**28.** D	